

(Extra)ordinary gauge mediation

Clifford Cheung,^{ab} A. Liam Fitzpatrick^a and David Shih^b

^a*Department of Physics, Harvard University,
Cambridge, MA 02138 U.S.A.*

^b*School of Natural Sciences, Institute for Advanced Study,
Princeton, NJ 08540 U.S.A.*

E-mail: cwcheung@fas.harvard.edu, fitzpatr@fas.harvard.edu, dshih@ias.edu

ABSTRACT: We study models of “(extra)ordinary gauge mediation,” which consist of taking ordinary gauge mediation and extending the messenger superpotential to include all renormalizable couplings consistent with SM gauge invariance and an R-symmetry. We classify all such models and find that their phenomenology can differ significantly from that of ordinary gauge mediation. Some highlights include: arbitrary modifications of the squark/slepton mass relations, small μ and Higgsino NLSP’s, and the possibility of having fewer than one effective messenger. We also show how these models lead naturally to extremely simple examples of direct gauge mediation, where SUSY and R-symmetry breaking occur not in a hidden sector, but due to the dynamics of the messenger sector itself.

KEYWORDS: Supersymmetry Breaking, Supersymmetry Phenomenology.

Contents

1. Introduction	2
1.1 Motivation	2
1.2 The phenomenology of (extra)ordinary gauge mediation	3
1.3 Minimal completions of gauge mediation	5
1.4 Outline	6
2. General aspects of (extra)ordinary gauge mediation	6
2.1 The models	6
2.2 MSSM soft masses	7
2.3 Doublet/triplet splitting and the MSSM soft masses	9
2.4 Doublet/triplet splitting and small μ	10
2.5 Small μ and the little hierarchy problem	12
2.6 Gauge coupling unification	13
3. Classification of models	15
3.1 Type I: theories with $\det m \neq 0$	15
3.2 Type II: theories with $\det \lambda \neq 0$	16
3.3 Type III: theories with $\det \lambda = \det m = 0$	17
4. Examples	17
4.1 Example 1: a family of type II models	17
4.2 Example 2: a family of type III models	21
5. Minimal completions of gauge mediation	24
5.1 Vacuum structure	24
5.2 More on R-symmetry breaking	25
5.3 Type II completions	27
5.4 Type III completions	30
A. The messenger mass matrix	33
A.1 Determinant identity	33
A.2 Messenger spectrum and the asymptotic behavior of N_{eff}	33
B. Renormalization methodology	34
C. Phenomenology of a Higgsino-like NLSP	37
C.1 Masses and mixings	37
C.2 Decay rates	38
C.3 Sparticle production at colliders	38

1. Introduction

1.1 Motivation

The LHC is coming, and the question on everyone’s mind is: what will we see? One reasonable guess is supersymmetry, probably still the most compelling candidate for physics beyond the standard model. The minimal incarnation of SUSY is the MSSM, but this is only an incomplete phenomenological framework. (For a nice review of the MSSM, see e.g. [1].) Soft SUSY breaking in the MSSM introduces ~ 100 new couplings in addition to those of the standard model, and in their most generic form, these new couplings give rise to serious flavor and CP problems. Thus, even if we discover the MSSM at the LHC, we will still have the main theoretical challenge ahead of us: explaining the origin of the MSSM parameters with an underlying model of SUSY breaking that is consistent with flavor and CP.

Gauge mediation [2–11] (see also [12–15] for reviews, and many relevant references) is a particularly attractive way of generating soft SUSY breaking in the MSSM. Not only does it solve the flavor and CP problems, but it is also calculable, predictive, and phenomenologically distinctive. Over the years, a great deal of work has been devoted to building complete models of gauge mediation, spurred by theoretical progress in constructing calculable examples [16, 17] of dynamical SUSY breaking [18]. As a result, there are now many viable models of gauge mediation, complete with detailed hidden sectors where SUSY is broken dynamically through strong gauge dynamics.

The study of the low-energy phenomenology of gauge mediation has proceeded in conjunction with these model-building efforts. Since the details of the hidden sector are often phenomenologically irrelevant,¹ people here have mostly relied on a simplified, incomplete framework known as “ordinary gauge mediation” (OGM), where the hidden sector is parameterized by a singlet field X which is a spurion for SUSY breaking,

$$\langle X \rangle = X + \theta^2 F, \tag{1.1}$$

(We will use X to denote both the superfield and the vev of its lowest component.) OGM also includes N vector-like pairs of messenger fields $\phi_i, \tilde{\phi}_i$, transforming in the $\mathbf{5} \oplus \bar{\mathbf{5}}$ representations under $SU(5) \supset G_{\text{SM}}$.² The messengers interact with X via Yukawa-like couplings

$$W = \lambda_{ij} X \phi_i \tilde{\phi}_j \tag{1.2}$$

where the sum on $i, j = 1, \dots, N$ is implicit. (Gauge indices are suppressed here and throughout.) Through (1.2) and the gauge interactions, the messengers communicate SUSY breaking from the hidden sector to the MSSM. The result is an MSSM spectrum with many distinctive features, some of which we will review later in this introduction.

¹This is not always the case, as was recently pointed out in [19].

²This is the simplest matter content consistent with gauge coupling unification. Other representations are also possible, including those that do not come in complete GUT multiplets [20], but we will not consider these here.

Given that much of the classic low-energy phenomenology of gauge mediation has been derived using the framework of OGM, it is important to ask (especially in the LHC era): is OGM truly representative of gauge mediation in general, or is it only one of many possible gauge mediation phenomenologies? In particular, how do things change if we deform or extend OGM in various directions?

In this paper, we would like to address these questions by studying a large family of extensions of OGM, obtained by generalizing (1.2) to include all renormalizable, gauge invariant couplings between the messengers and any number of singlet fields. Since these models extend OGM into a wider parameter space, yet they are no less “ordinary” by any sensible measure (i.e. they are renormalizable and are not forbidden by any symmetries or experimental constraints), we will refer to them as models of “(extra)ordinary gauge mediation” (EOGM). In the following sections, we will present an in-depth study of the phenomenology of EOGM, and we will see that it can differ in interesting ways from that of OGM.

1.2 The phenomenology of (extra)ordinary gauge mediation

Now let us describe our EOGM models and their phenomenology in more detail. We start with the most general renormalizable, gauge invariant superpotential describing the couplings between the messengers and any number of singlets X_k :

$$W = (\lambda_{ij}^{(k)} X_k + M_{ij}) \phi_i \tilde{\phi}_j = (\lambda_{2ij}^{(k)} X_k + M_{2ij}) \ell_i \tilde{\ell}_j + (\lambda_{3ij}^{(k)} X_k + M_{3ij}) q_i \tilde{q}_j \quad (1.3)$$

where in the second equation of (1.3), we have decomposed $\phi_i, \tilde{\phi}_i$ into their SU(2) doublet and SU(3) triplet components, $\ell_i, \tilde{\ell}_i$ and q_i, \tilde{q}_i , respectively. We emphasize that doublet/triplet splitting in (1.3) is similar in spirit to the doublet/triplet splitting that already happens in SUSY GUT embeddings of the MSSM (indeed they may very well have the same origin), so there is really no reason not to consider the most general form of (1.3).

In fact, this model can be reduced to a model with only one singlet, through the following trivial observation. Through a unitary transformation, we can always rotate the singlet fields so that only one of them, call it X , acquires a SUSY-breaking F-component vev as in (1.1). Then the remaining singlets only have scalar component vevs, $\langle X_k \rangle = X_k$, and since we are only interested in the tree-level messenger mass matrix, we are free to substitute these into the superpotential (1.3). This reduces it to the form

$$W = (\lambda_{ij} X + m_{ij}) \phi_i \tilde{\phi}_j = (\lambda_{2ij} X + m_{2ij}) \ell_i \tilde{\ell}_j + (\lambda_{3ij} X + m_{3ij}) q_i \tilde{q}_j \quad (1.4)$$

In other words, we have shown that the most general EOGM model is simply OGM plus arbitrary supersymmetric mass terms for the messengers.

Surprisingly, while there are many examples in the literature of OGM deformed by mass terms (including many of the original models of gauge mediation [4–8], some more modern models [21–25], and most recently many of the models based on [26]), the phenomenology

of these models has not been explored in any systematic way.³ In this paper, we will take the first steps in this direction.

To simplify our analysis, and because it has some distinctive and desirable consequences, we will limit our study to models possessing a non-trivial $U(1)_R$ symmetry, which is only broken spontaneously by the vev of X (1.1).⁴ We will show that in a general EOGM model with an R-symmetry, the soft masses at the messenger scale are given by a simple generalization of the usual OGM formulae

$$M_r = \frac{\alpha_r}{4\pi} \Lambda_G, \quad m_{\tilde{f}}^2 = 2 \sum_{r=1}^3 C_{\tilde{f}}^r \left(\frac{\alpha_r}{4\pi} \right)^2 \Lambda_G^2 N_{\text{eff},r}^{-1} \quad (1.5)$$

Here $\Lambda_G \sim F/X$ sets the overall scale of the soft masses, $N_{\text{eff},2}$ and $N_{\text{eff},3}$ can be thought of as “effective” doublet and triplet messenger numbers, and $N_{\text{eff},1}^{-1} \equiv \frac{3}{5} N_{\text{eff},2}^{-1} + \frac{2}{5} N_{\text{eff},3}^{-1}$.⁵ In general, $N_{\text{eff},2}$ and $N_{\text{eff},3}$ depend on all the doublet and triplet parameters of the model, respectively:

$$N_{\text{eff},r} \equiv N_{\text{eff}}(X, m_r, \lambda_r) \quad (r = 2, 3) \quad (1.6)$$

and they take values between 0 and N inclusive. (The full formula for N_{eff} can be found in section 2.) Note that the special case of OGM corresponds to $N_{\text{eff},2} = N_{\text{eff},3} = N$ — the messenger numbers in this case are equal and are independent of all the couplings.

The effective messenger numbers play an important role in determining the low-energy phenomenology of EOGM. In particular, (1.6) implies that doublet/triplet splitting can lead to different effective messenger numbers for doublets and triplets (unlike in OGM), and this in turn can have a large, qualitative effect on the spectrum. Some specific ways in which EOGM can deviate from OGM include:

1. *Modified relations between squark and slepton masses.* Typically, in gauge mediation, the squark mass-squareds are always much larger than the slepton mass-squareds, since $\alpha_3 \gg \alpha_2, \alpha_1$. However, by making $N_{\text{eff},3} \gg N_{\text{eff},2}$, the sfermion masses can be squashed together, as can be seen from (1.5).
2. *The possibility for small μ and Higgsino NLSPs in a large portion of parameter space.* A more subtle consequence of having different doublet and triplet messenger numbers is that this can lead to small μ through a cancellation in the running of $m_{H_u}^2$ [27, 28]. Aside from its possible implications for the little hierarchy problem, small μ in gauge mediation is interesting because it implies that the NLSP is a Higgsino-like neutralino.

³Perhaps one reason for this is that generic multi-messenger models are problematic, as they generally have tachyonic one-loop slepton masses coming from contractions of the hypercharge D-terms. (We thank M. Dine for pointing out this effect to us.) We will discuss this problem — and how we get around it — in more detail in section 2.

⁴These models always possess a trivial R-symmetry under which $R(X) = 0$ and $R(\phi_i) = R(\tilde{\phi}_i) = 1$. The $U(1)_R$ we are imposing on (1.4) is in addition to this, and it results in various selection rules on the couplings m_{ij}, λ_{ij} .

⁵The rest of the notation is as in [12]. In particular, $r = 1, 2, 3$ labels the SM gauge groups $U(1), SU(2)$ and $SU(3)$, respectively; \tilde{f} labels an MSSM sfermion field; and $C_{\tilde{f}}^r$ is the quadratic Casimir of \tilde{f} in the gauge group r .

This novel scenario has not been studied much in the past (see however [25, 29–32]), presumably because in OGM the NLSP is always either the bino or the stau.

3. *Effective messenger number less than one.* In the space of EOGM models, one can achieve $N_{\text{eff}} < 1$, which is obviously never possible in OGM where $N_{\text{eff}} = N$. This is interesting, as it allows the gauginos to be lighter than in any OGM scenario. Lighter gluinos, in particular, could significantly enhance sparticle production at the LHC, relative to standard OGM rates.
4. *Gauge coupling unification.* We will see that in these models, the R-symmetry allows for gauge coupling unification to be maintained without tuning of parameters, even with different effective numbers of doublet and triplet messengers. This is a crucial difference between these models and those of [27, 28], where additional doublet and/or triplet fields were put in by hand to ensure unification.

Finally, let us mention one aspect of the spectrum that does not change between OGM and EOGM models (with an R-symmetry). According to (1.5), the gaugino masses always obey the GUT relations in these models, $M_1 : M_2 : M_3 = \alpha_1 : \alpha_2 : \alpha_3$, regardless of the amount of doublet/triplet splitting. As we will see in the next section, this is a direct consequence of imposing a non-trivial R-symmetry on the superpotential (1.4) under which $R(X) \neq 0$. In more general models without such an R-symmetry, even the gaugino mass relations can be modified arbitrarily through doublet/triplet splitting.

1.3 Minimal completions of gauge mediation

In addition to exploring the phenomenology of gauge mediation, there is another, more formal motivation for studying models of the form (1.4): the goal of finding simple examples of “direct gauge mediation,” i.e. models in which the messengers are also part of the SUSY breaking sector. Indeed, our EOGM models can be trivially completed into generalized O’Raifeartaigh models of the kind discussed recently in [33], simply by adding $\delta W = FX$ to (1.4):

$$W = \lambda_{ij} X \phi_i \tilde{\phi}_j + m_{ij} \phi_i \tilde{\phi}_j + FX \tag{1.7}$$

As we will see, the R-symmetry guarantees that the tree-level scalar potential has a pseudo-moduli space of SUSY-breaking local minima, located at $\phi = \tilde{\phi} = 0$ and $|X|$ in some window. At one-loop, a Coleman-Weinberg potential is generated on the pseudo-moduli space, and the minima of this potential (if they exist) are SUSY-breaking vacua of the theory.

In order for these models to be phenomenologically viable, the R-symmetry must be spontaneously broken in the vacuum (otherwise the gauginos cannot obtain soft masses). We will see that such R-symmetry breaking minima of the CW potential can exist in the parameter space of these models, because there are typically fields with R-charge $R \neq 0, 2$ [33]. Therefore, these models can serve as extremely compact examples of direct gauge mediation, which are complete in the sense that the sources of SUSY and R-symmetry breaking are included. Note that these models are *not* examples of dynamical

SUSY breaking, nor do they explain the origin of μ and $B\mu$. However, they do provide a minimal framework in which these issues can be further explored.

1.4 Outline

The outline of our paper is as follows. In section 2 we will discuss some general aspects of EOGM, including: formulae for $N_{\text{eff}}(X, m, \lambda)$ and the MSSM soft masses; a discussion of doublet/triplet splitting and its effects; and the issue of gauge coupling unification. In section 3 we will introduce a classification of EOGM models. We will see that the models fall into three distinct categories which have qualitatively different phenomenology. In section 4 we will analyze in detail the phenomenology of some simple examples of EOGM models and show how some of the general features discussed in section 2 can be realized. Finally, section 5 contains an analysis of the minimal completions (1.7).

In appendix A, we prove some useful results about the mass matrix of the messengers, which have implications for the MSSM soft SUSY-breaking terms. Appendix B has a discussion of our treatment of the MSSM RGEs, a careful understanding of which is important for obtaining accurate low-energy MSSM spectra. In appendix C, there are some useful formulae for the neutralino and chargino mass matrices in the small μ limit, as well as a very preliminary discussion of the collider phenomenology of Higgsino NLSPs.

2. General aspects of (extra)ordinary gauge mediation

2.1 The models

In this section, we would like to study general aspects of the phenomenology of EOGM models. As discussed in the introduction, the models consist of a singlet X and N messengers $\phi_i, \tilde{\phi}_i$ transforming in the $\mathbf{5} \oplus \bar{\mathbf{5}}$ representation of $SU(5) \supset G_{\text{SM}}$. Through some unspecified dynamics in the hidden sector, X acquires a SUSY- and R-symmetry-breaking vev, $\langle X \rangle = X + \theta^2 F$. The couplings between X and the messengers are described by the most general superpotential consistent with renormalizability, SM gauge invariance, and a non-trivial R-symmetry:

$$W = \mathcal{M}_{ij}(X)\phi_i\tilde{\phi}_j = (\lambda_{ij}X + m_{ij})\phi_i\tilde{\phi}_j \tag{2.1}$$

where $\mathcal{M}_{ij}(X) = \lambda_{ij}X + m_{ij}$ is the messenger mass matrix, and the R-symmetry means that the couplings in this mass matrix must obey a set of selection rules.⁶ Let us now describe these selection rules in more detail. For the time being, we will assume for simplicity that the couplings in (2.1) respect the full $SU(5)$ invariance; in section 2.3 and beyond, we will consider the effect of doublet/triplet splitting in detail.

Suppose that (2.1) respects a non-trivial R-symmetry under which the fields transform with R-charges $R(X) \neq 0, R(\phi_i), R(\tilde{\phi}_i)$. (This will be the case for all the models studied

⁶Note that although we are imposing this R-symmetry on the messenger superpotential, it could actually be an accidental symmetry of the underlying, strongly-coupled gauge theory which presumably dynamically generates all the mass scales in (1.4) (and in which X and/or the messengers could be composite fields). This is precisely what happens, for instance, in massive SQCD in the free-magnetic phase [26].

in this paper.) Then the selection rules take the form

$$\begin{aligned} \lambda_{ij} \neq 0 & \quad \text{only if} \quad R(\phi_i) + R(\tilde{\phi}_j) = 2 - R(X) \\ m_{ij} \neq 0 & \quad \text{only if} \quad R(\phi_i) + R(\tilde{\phi}_j) = 2 \end{aligned} \quad (2.2)$$

since W must always have definite R-charge $R(W) = 2$,

These selection rules, and the R-symmetry more generally, have many important consequences which we will explore in the following subsections, starting with the spectrum of MSSM soft masses. Most of these consequences stem from a non-trivial identity satisfied by the messenger mass matrix,

$$\det \mathcal{M} = X^n G(m, \lambda), \quad n = \frac{1}{R(X)} \sum_{i=1}^N (2 - R(\phi_i) - R(\tilde{\phi}_i)), \quad (2.3)$$

where $G(m, \lambda)$ is some function of the couplings. This identity follows directly from the selection rules (2.2); for a straightforward proof, see appendix A. Note that in this identity, n must be an integer satisfying $0 \leq n \leq N$, since $\det(\lambda X + m)$ is a degree N polynomial in X .

Although we have allowed $R(X)$ to take any non-zero value in the discussion of the R-symmetry so far, it turns out that not all of these R-symmetries are distinct. In fact, if the model is invariant under an R-symmetry with $R(X) \neq 0$, then it must be invariant under a continuous family of *equivalent* R-symmetries parametrized by arbitrary $R(X) \in \mathbb{R}$. These are obtained by mixing the R-symmetry with the trivial $U(1)_R$ that is always respected by (2.1), under which $R(X) = 0$, $R(\phi_i) = R(\tilde{\phi}_i) = 1$. (As a consistency check, note that the formula for n in (2.3) remains invariant under this mixing of R-symmetries.) In particular, we can always use this to set

$$R(X) = 2 \quad (2.4)$$

without loss of generality. Henceforth, we will assume this implicitly in the paper. This will turn out to be a convenient choice in section 5, where we analyze the “complete” models obtained by perturbing (2.1) by the SUSY-breaking linear term $\delta W = FX$.

2.2 MSSM soft masses

It is straightforward to derive formulae for the running gaugino and sfermion soft masses at the messenger scale, by generalizing the wavefunction renormalization technique of [34]. For the gaugino masses we find (using the determinant identity (2.3))

$$M_r = \frac{\alpha_r}{4\pi} \Lambda_G, \quad \Lambda_G = F \partial_X \log \det \mathcal{M} = \frac{nF}{X} \quad (2.5)$$

while the sfermion masses are given by

$$m_{\tilde{f}}^2 = 2 \sum_{r=1}^3 C_{\tilde{f}}^r \left(\frac{\alpha_r}{4\pi} \right)^2 \Lambda_S^2, \quad \Lambda_S^2 = \frac{1}{2} |F|^2 \frac{\partial^2}{\partial X \partial X^*} \sum_{i=1}^N (\log |\mathcal{M}_i|^2)^2 \quad (2.6)$$

where \mathcal{M}_i denote the eigenvalues of \mathcal{M} . (The rest of the notation is described in the introduction.) In these formulas, the gauge couplings α_r are all evaluated at the messenger

scale. In order to find the physical spectrum, one must of course run everything down to the weak scale. Our procedure for this is described in appendix B.

The soft masses (2.5) and (2.6) are generalizations of well-known OGM formulae. (See e.g. [12], whose conventions we largely adhere to in this paper.) By analogy with OGM, it is useful to define the “effective messenger number” to be

$$N_{\text{eff}}(X, m, \lambda) \equiv \frac{\Lambda_G^2}{\Lambda_S^2} = \left[\frac{1}{2n^2} |X|^2 \frac{\partial^2}{\partial X \partial X^*} \sum_{i=1}^N \left(\log \frac{|\mathcal{M}_i|^2}{\mu^2} \right)^2 \right]^{-1} \quad (2.7)$$

In OGM, $N_{\text{eff}} = N$, but more generally it is a continuous function of the couplings taking values between 0 and N inclusive.

A fact that will be useful in later sections is that N_{eff} simplifies somewhat in the asymptotic limits $X \rightarrow 0, \infty$. In appendix A, we derive formulas for N_{eff} in these limits. Here let us simply highlight two features of these formulas that we will need later. First of all, the asymptotic values of N_{eff} are independent of all the parameters,

$$\lim_{X \rightarrow 0, \infty} N_{\text{eff}}(X, m, \lambda) = \text{const.} \quad (2.8)$$

Second, the asymptotic values of N_{eff} satisfy the inequalities

$$\frac{n^2}{n^2 - (N - r_m - 1)(2n - N + r_m)} \leq N_{\text{eff}}(X \rightarrow 0) \leq N - r_m \quad (2.9)$$

and

$$\frac{n^2}{r_\lambda + (r_\lambda - n)^2} \leq N_{\text{eff}}(X \rightarrow \infty) \leq \frac{n^2}{r_\lambda + \frac{(r_\lambda - n)^2}{(N - r_\lambda)}} \quad (2.10)$$

where we have introduced the notation

$$r_\lambda \equiv \text{rank } \lambda, \quad r_m \equiv \text{rank } m \quad (2.11)$$

This notation will also prove to be useful below.⁷

Finally, let us conclude this subsection by pointing out two effects that we have ignored in writing down our formulae (2.5), (2.6) for the MSSM soft masses. The first is the effect of multiple messenger scales. These can modify the formulae for the soft masses through RG evolution, but in general this is a small effect. Below, in our more quantitative analysis of specific examples, we will fully account for the multiple messenger thresholds. The second effect we are ignoring is the contribution to Λ_G, Λ_S from higher-order corrections in F/M_{mess}^2 , where M_{mess} is the (lightest) messenger scale. These cannot be extracted from wavefunction renormalization, but instead require a full Feynman diagram calculation. In the following we will assume implicitly that $F \ll M_{\text{mess}}^2$, in which case these corrections are negligible.

⁷Note that when $r_\lambda = N$, the upper bound on $N_{\text{eff}}(X \rightarrow \infty)$ in (2.10) no longer makes sense. However, as we will discuss more fully in section 3.2 below, in this case one always has $N_{\text{eff}}(X \rightarrow \infty) = N$.

2.3 Doublet/triplet splitting and the MSSM soft masses

So far, we have assumed for simplicity that the couplings in the superpotential (2.1) respect the full SU(5) gauge symmetry. However, the most general superpotential need only respect the SM gauge symmetry; thus we are led to consider

$$W = (\lambda_{2ij}X + m_{2ij})\ell_i\tilde{\ell}_j + (\lambda_{3ij}X + m_{3ij})q_i\tilde{q}_j \quad (2.12)$$

where ℓ , $\tilde{\ell}$ and q , \tilde{q} denote SU(2) doublets and SU(3) triplets, respectively. In this subsection, we would like to describe the effect of doublet/triplet splitting on the MSSM soft masses. Throughout, we will assume that the doublet and triplet messengers have the same R-charge assignments. As a result, the doublet and triplet messenger mass matrices will always have the same structure and will both obey (2.3) with the same n and the same function G .

As discussed in the introduction, doublet/triplet splitting has little effect on the MSSM soft masses in OGM. Thus, even allowing for arbitrary doublet/triplet splitting, OGM leads to very distinctive relations among the gaugino and the sfermion masses. In EOGM, the relations amongst the gaugino masses are still preserved,

$$M_1 : M_2 : M_3 = \alpha_1 : \alpha_2 : \alpha_3 \quad (2.13)$$

even with an arbitrary amount of doublet/triplet splitting. This follows from (2.5), according to which Λ_G is independent of the couplings and only depends on the integer n (which is the same between the doublets and triplets). Let us emphasize that this is a direct consequence of imposing on the model (2.1) a non-trivial R-symmetry under which $R(X) \neq 0$; if we abandon this symmetry, then the GUT relations for the gaugino masses need no longer hold.⁸

Next let us consider the sfermion masses. Here we have, instead of (2.6):

$$m_{\tilde{f}}^2 = 2 \sum_{r=1}^3 C_{\tilde{f}}^r \left(\frac{\alpha_r}{4\pi}\right)^2 \Lambda_{S_r}^2 \quad (2.14)$$

with

$$\Lambda_{S_2}^2 = \Lambda_G^2 N_{\text{eff}}(X, m_2, \lambda_2)^{-1}, \quad \Lambda_{S_3}^2 = \Lambda_G^2 N_{\text{eff}}(X, m_3, \lambda_3)^{-1} \quad (2.15)$$

and $\Lambda_{S_1}^2 = \frac{2}{5}\Lambda_{S_3}^2 + \frac{3}{5}\Lambda_{S_2}^2$. Thus, the mass relations amongst the sfermions can be arbitrarily modified through doublet/triplet splitting. In particular, by taking $N_{\text{eff},3} \gg N_{\text{eff},2}$, the squark and slepton masses can be brought closer together than in OGM (where typically $m_{\tilde{t}}/m_{\tilde{e}_R} \sim 7-10$). This could be helpful for solving the “little hierarchy problem” of OGM, where – independent of the LEP bound on the Higgs mass – the squarks must be at least 700 GeV given the experimental lower bound of ~ 100 GeV on the selectron mass.

⁸M. Dine has pointed out to us that in EOGM models which do not have an R-symmetry, doublet/triplet splitting can in general lead to dangerous $\mathcal{O}(1)$ CP-violating phases in the gaugino masses. Thus, avoiding these phases could be viewed as another motivation for imposing a non-trivial R-symmetry on the space of EOGM models.

Note that in writing down (2.14), we have not included the dangerous contributions to the sfermion masses coming from contractions of the messenger hypercharge D-terms $D = g'(\phi^\dagger Y_\phi \phi - \tilde{\phi}^T Y_\phi \tilde{\phi}^*)$ [4]. If present, these contributions cause either the right or the left-handed sleptons to become tachyonic, because they are not positive definite, and they appear already at one-loop in the gauge interactions. They are absent in OGM, but unfortunately not in a generic EOGM model with arbitrary doublet/triplet splitting. In order to forbid these D-term contributions, we will impose on all the models we study in this paper the “messenger parity” symmetry proposed in [24]

$$\phi \rightarrow U^* \tilde{\phi}^*, \quad \tilde{\phi} \rightarrow \tilde{U} \phi^*, \quad V \rightarrow -V \tag{2.16}$$

where V stands for the SM gauge superfields, and U and \tilde{U} are some $N \times N$ unitary matrices. This is a symmetry of the messenger Lagrangian provided that

$$\mathcal{M}^\dagger = U^\dagger \mathcal{M} \tilde{U}, \quad (\lambda F)^\dagger = U^\dagger (\lambda F) \tilde{U} \tag{2.17}$$

and it forces the dangerous hypercharge D-term contributions to the sfermion masses to vanish, since they are odd under it. Of course, this parity is explicitly broken by the MSSM matter fields, so there will still be hypercharge D-term contributions from loops involving both MSSM and messenger fields. However, these only enter in at three-loops and higher, so they will be negligible compared to the two-loop mass-squareds shown above.

2.4 Doublet/triplet splitting and small μ

In this subsection, we would like to analyze a more subtle effect of doublet/triplet splitting on the MSSM spectrum, namely the possibility of having small μ through a cancellation in the running of $m_{H_u}^2$. This “focussing” effect was first pointed out in [27, 28].

To begin, let us recall that electroweak symmetry-breaking in the MSSM specifies μ (up to a sign) in terms of the soft masses at the electroweak scale. At large $\tan\beta$, the relation is approximately

$$\mu^2 \approx -\frac{1}{2}m_Z^2 - m_{H_u}^2(m_{\tilde{t}}) \tag{2.18}$$

where the value of $m_{H_u}^2$ at $Q = m_{\tilde{t}}$ is approximately given by its gauge mediation value (2.14) plus the dominant contribution to the one-loop running coming from stop loops:

$$m_{H_u}^2(m_{\tilde{t}}) \approx m_{H_u}^2 - \frac{3}{4\pi^2} y_t^2 m_{\tilde{t}}^2 \log \frac{M_{\text{mess},3}}{m_{\tilde{t}}} \tag{2.19}$$

From (2.18)–(2.19), we see that μ will be small if a cancellation can be arranged between the two terms on the r.h.s. above [27, 28]. According to the general formulae (2.14),

$$m_{H_u}^2 \propto \frac{3}{4} \frac{\alpha_2 (M_{\text{mess},2})^2}{N_{\text{eff},2}}, \quad m_{\tilde{t}}^2 \propto \frac{4}{3} \frac{\alpha_3 (M_{\text{mess},3})^2}{N_{\text{eff},3}} \tag{2.20}$$

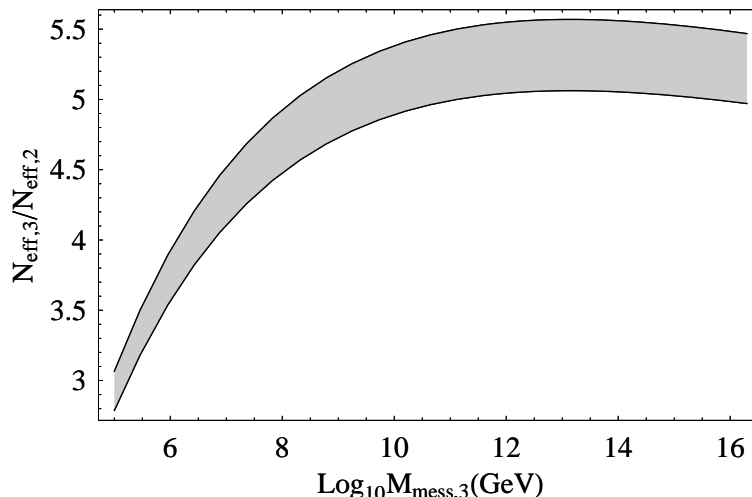


Figure 1: A plot of the range of $N_{\text{eff},3}/N_{\text{eff},2}$ where at least a one part in ten cancellation occurs in the running of $m_{H_u}^2$. The range is plotted vs. the triplet messenger scale $M_{\text{mess},3}$; the doublet scale $M_{\text{mess},2}$ was fixed to be 500 TeV.

so a cancellation can occur if $N_{\text{eff},3} \gg N_{\text{eff},2}$, i.e. if the colored and uncolored sparticle masses are squashed together.⁹

We can elaborate upon this point more quantitatively. If we define the degree of cancellation,

$$\begin{aligned} \eta &= -\frac{m_{H_u}^2(m_{\tilde{t}})}{m_{H_u}^2} \\ &\approx \frac{4y_t^2}{3\pi^2} \frac{\alpha_3(M_{\text{mess},3})^2}{\alpha_2(M_{\text{mess},2})^2} \frac{N_{\text{eff},2}}{N_{\text{eff},3}} \log \frac{M_{\text{mess},3}}{m_{\tilde{t}}} - 1 \end{aligned} \quad (2.21)$$

and require $0 \leq \eta \leq 0.1$ (the lower bound is the requirement of electroweak symmetry breaking), we obtain a range in $N_{\text{eff},3}/N_{\text{eff},2}$ for a given set of triplet and doublet messenger scales $M_{\text{mess},3}$, $M_{\text{mess},2}$. To illustrate this, we have plotted in figure 1 an example of this range as a function of $M_{\text{mess},3}$, for $M_{\text{mess},2} = 500$ TeV (and $y_t \approx 1$, $m_{\tilde{t}} \approx 1$ TeV). From this plot, we can glean a few general facts about what it takes to achieve a cancellation in the running of $m_{H_u}^2$.

First, we see that $N_{\text{eff},3}/N_{\text{eff},2}$ cannot be too large, otherwise $m_{H_u}^2(m_{\tilde{t}})$ is positive and electroweak symmetry breaking does not even occur. Second, we see that generally one needs at least three times more effective triplet messengers than doublet messengers in order to get a significant cancellation. A corollary of this is that in OGM one never gets a cancellation in the running of $m_{H_u}^2$, since there $N_{\text{eff},3} = N_{\text{eff},2} = N$. Indeed, in OGM

⁹In [27, 28], different numbers of OGM doublet and triplet messengers were put in by hand, and additional heavy doublets and/or triplets were included in an ad hoc fashion just for the sake of gauge coupling unification. As we will show below, our models are more natural, in that the R-symmetry guarantees the presence of heavy messengers at the correct scales for unification, even when $N_{\text{eff},3} \neq N_{\text{eff},2}$ and there is a large amount of doublet/triplet splitting.

one typically has $|\mu| \gtrsim 1 \text{ TeV}$ (given the LEP bound on the Higgs mass), and there is an absolute lower bound of $\sim 350 \text{ GeV}$ on $|\mu|$. By contrast, in EOGM with sufficiently many messengers it is possible to get $|\mu|$ arbitrarily small even while keeping fixed $m_{\tilde{\tau}} \gtrsim 1 \text{ TeV}$ to satisfy the LEP bound on the Higgs mass.

Small μ is very interesting because, among other reasons, it implies a Higgsino-like neutralino NLSP. (Formulae for the Higgsino fractions of the lightest neutralinos and charginos in the small μ limit can be found in appendix C.) Although the possibility of Higgsino NLSPs in gauge mediation has been considered before, for instance in [25, 29–32], this scenario has not been given much attention, essentially because of the theoretical bias from OGM where the NLSP is always either the bino or the right-handed stau. Needless to say, the collider phenomenology of Higgsino NLSPs can be quite different from that of bino or stau-like NLSPs. For instance, a Higgsino NLSP will have a suppressed branching fraction to $\gamma + \tilde{G}$ and enhanced branching fractions to $h + \tilde{G}$ and $Z + \tilde{G}$. Consequently, the classic $\gamma\gamma + \cancel{E}_T$ channel might no longer be the preferred discovery mode for gauge mediation, if the Higgsino is the NLSP.

In our examples below, we will see that in models with sufficiently many messengers, Higgsino NLSPs can occur in a wide range of the EOGM parameter space. Therefore, we would argue that this scenario deserves more study. Some preliminary remarks on the phenomenology of Higgsino NLSPs are contained in appendix C. A detailed analysis would take us too far afield in this paper, so we will leave this work for a future publication [35].

2.5 Small μ and the little hierarchy problem

Another reason small μ is interesting is because of its implications for naturalness and the “little hierarchy problem.” The little hierarchy problem is usually cast in terms of the amount of cancellation or fine-tuning required in (2.18) between the supersymmetric μ parameter and the soft SUSY-breaking $m_{H_u}^2$ parameter, in order to achieve the observed value of m_Z^2 . The amount of fine tuning with respect to a coupling λ is often quantified in terms of the Barbieri-Giudice measure [36],

$$\Delta_\lambda(m_Z^2) = \left| \frac{\partial \log m_Z^2}{\partial \log \lambda} \right| \tag{2.22}$$

That is, Δ_λ^{-1} corresponds to the percent fine-tuning in the parameter λ required to achieve the observed value of m_Z^2 . For instance, the fine tuning associated with the μ parameter is

$$\Delta_{\mu^2}(m_Z^2) = \left| \frac{\partial \log m_Z^2}{\partial \log \mu^2} \right| = \frac{2\mu^2}{m_Z^2} \tag{2.23}$$

As mentioned in the previous subsection, in OGM one typically has $|\mu| \gtrsim 1 \text{ TeV}$ because of the LEP bound on the Higgs mass. Thus OGM — like much of the MSSM parameter space — has a little hierarchy problem in that it is fine-tuned to at least the percent level with respect to μ . (For a recent, more detailed discussion of the fine-tuning problem in OGM, see e.g. [37].)

Now let us contrast this with the situation in EOGM. We have seen in the previous subsection that, by having different effective doublet and triplet messenger numbers, it

is possible in EOGM to have $\mu \sim 100$ GeV even with TeV scale stop masses. Thus, the fine-tuning with respect to μ in EOGM can be improved to $\mathcal{O}(10\%)$ or better, and this puts us one step closer to solving the little hierarchy problem.

Of course, the route to small μ in EOGM is through a partial cancellation between the gauge mediation contribution to $m_{H_u}^2$ at the messenger scale, and the radiative corrections to $m_{H_u}^2$ coming from RG evolution down to the weak scale. Thus one might wonder whether the reduction in fine-tuning with respect to μ is merely being compensated for by an increased fine-tuning with respect to other parameters responsible for the cancellation. In fact, the situation can be better than it seems, because the cancellation depends on $N_{\text{eff},3}/N_{\text{eff},2}$, and if these are taking their asymptotic values at $X \rightarrow 0$ or $X \rightarrow \infty$, then they are actually *insensitive to the couplings*, as noted in (2.8).

To make this a bit more precise, let us estimate the fine tuning with respect to the other parameters of the model using (2.18)–(2.21) and the Barbieri-Giudice measure. This gives

$$\begin{aligned} \Delta_\lambda(m_Z^2) &\approx \left| \frac{2\lambda}{m_Z^2} \frac{\partial m_{H_u}^2(m_{\tilde{t}})}{\partial \lambda} \right| \\ &\approx \left| \frac{2\lambda}{m_Z^2} \partial_\lambda \left[\left(\frac{nF}{X} \right)^2 \left(\frac{3}{2} \left(\frac{\alpha_2}{4\pi} \right)^2 N_{\text{eff},2}^{-1} - \frac{3}{4\pi^2} y_t^2 \times \frac{8}{3} \left(\frac{\alpha_3}{4\pi} \right)^2 N_{\text{eff},3}^{-1} \log \frac{M_{\text{mess},3}}{m_{\tilde{t}}} \right) \right] \right| \end{aligned} \tag{2.24}$$

If we assume that $N_{\text{eff},2}$ and $N_{\text{eff},3}$ are given by their asymptotic values as in (2.8), then they are essentially constants. Then the fine-tuning (2.24) will be negligible with respect to most of the parameters of the model; the only ones that matter are $M_{\text{mess},3}$, y_t , α_2 and α_3 , and F/X . The Barbieri-Giudice measure for these are either the same or smaller than in a theory without focussing. Therefore, we conclude that the overall amount of fine tuning is reduced in these models, due to the insensitivity of the asymptotic values of N_{eff} (and hence the amount of cancellation) to the model parameters.

2.6 Gauge coupling unification

We have seen how doublet/triplet splitting in EOGM can have interesting effects on the MSSM spectrum. However, all these results would be significantly less interesting if they required an amount of doublet/triplet splitting that ruined the successful unification of the gauge couplings seen in the MSSM. In this subsection, we would like to analyze this issue in detail. We will see that because of the R-symmetry, the sensitivity of the running of the gauge couplings to doublet/triplet splitting is significantly reduced, meaning that it is possible to achieve all the effects described in the previous subsections without sacrificing unification.

To begin, let us consider the one-loop RG evolution of the gauge couplings up to the GUT scale m_{GUT} . After passing through all the individual doublet and triplet messenger thresholds, one finds that the value of the gauge couplings at m_{GUT} depends only on the “average” doublet and triplet messenger scales,

$$\overline{\mathcal{M}}_{2,3} \equiv (\det \mathcal{M}_{2,3})^{1/N} \tag{2.25}$$

More precisely, one finds

$$\alpha_r^{-1}(m_{\text{GUT}}) = \alpha_r^{-1}(m_Z) + \frac{b_r}{2\pi} \log \frac{m_{\text{GUT}}}{m_Z} - \frac{N}{2\pi} \log \frac{m_{\text{GUT}}}{\overline{\mathcal{M}}_r} \quad (2.26)$$

for $r = 1, 2, 3$. Here $b_r = (-\frac{33}{5}, -1, 3)$ denotes the MSSM one-loop β functions, and $\overline{\mathcal{M}}_1 \equiv (\overline{\mathcal{M}}_2)^{3/5}(\overline{\mathcal{M}}_3)^{2/5}$. Note that the first two terms in (2.26) correspond to the value of the MSSM gauge couplings at the GUT scale. As is well-known, these unify to a high degree of precision (more on this in the next paragraph), with a common value at the GUT scale given by

$$\alpha_r^{-1}(m_Z) + \frac{b_r}{2\pi} \log \frac{m_{\text{GUT}}}{m_Z} \approx \alpha_{\text{GUT,MSSM}}^{-1} \approx 24.3 \quad (2.27)$$

Combining (2.26) and (2.27), we conclude that when $\overline{\mathcal{M}}_2 = \overline{\mathcal{M}}_3$, unification occurs precisely as in the MSSM. Furthermore, the determinant identity (2.3) tells us that $\overline{\mathcal{M}}_{2,3} = (X^n G(m_{2,3}, \lambda_{2,3}))^{1/N}$, and as we will see in the next section, the function G is generally independent of some subset of the couplings. Therefore, with this subset of couplings, we can still achieve an arbitrary amount of doublet/triplet splitting, while preserving the same precision of unification seen in the MSSM.

For the sake of completeness, let us also work out how much splitting between $\overline{\mathcal{M}}_2, \overline{\mathcal{M}}_3$ can be tolerated without spoiling unification. A commonly used measure of unification (see e.g. [38, 39]) is the quantity

$$B \equiv \frac{\alpha_2^{-1}(m_Z) - \alpha_3^{-1}(m_Z)}{\alpha_1^{-1}(m_Z) - \alpha_2^{-1}(m_Z)} \quad (2.28)$$

By assuming unification and running the gauge couplings down from the GUT scale, one obtains a prediction for B that can be compared with experiment. The one-loop MSSM prediction is $B = \frac{b_3 - b_2}{b_2 - b_1} = \frac{5}{7}$, and this agrees with experiment to approximately 5% accuracy, where the bulk of the uncertainty comes from the unknown GUT and MSSM thresholds. In our models, it follows from setting $\alpha_1(m_{\text{GUT}}) = \alpha_2(m_{\text{GUT}}) = \alpha_3(m_{\text{GUT}})$ in (2.26) that

$$B = \frac{(b_3 - b_2) \log \left(\frac{m_{\text{GUT}}}{m_Z} \right) + N (\log \overline{\mathcal{M}}_3 - \log \overline{\mathcal{M}}_2)}{(b_2 - b_1) \log \left(\frac{m_{\text{GUT}}}{m_Z} \right) - \frac{2}{5} N (\log \overline{\mathcal{M}}_3 - \log \overline{\mathcal{M}}_2)} \quad (2.29)$$

Setting $N = 0$ or $\overline{\mathcal{M}}_3 = \overline{\mathcal{M}}_2$ in (2.29) gives the one-loop MSSM value. If we are to deviate no more than 5% from this, then we require

$$N \left| \log \frac{\overline{\mathcal{M}}_3}{\overline{\mathcal{M}}_2} \right| \lesssim 5 \quad (2.30)$$

where we have used $\log(m_{\text{GUT}}/m_Z) \approx 33$. According to this inequality, the amount of splitting in the average messenger scales that we are allowed to tolerate depends sensitively on the messenger number N . For $N = 1$ we can split the average messenger scales by as much as a factor of 100. But for $N = 5$ we can only tolerate a factor of a few. However, let us reiterate that it is possible to have an arbitrary amount of doublet/triplet splitting yet still keep $\overline{\mathcal{M}}_3 \approx \overline{\mathcal{M}}_2$, because of the determinant identity (2.3).

Finally, let us see what the requirement of perturbativity up to the GUT scale looks like in these models. Taking $\overline{\mathcal{M}}_2 \approx \overline{\mathcal{M}}_3 \equiv \overline{\mathcal{M}}$ as required by (2.30), and demanding that $\alpha_r^{-1}(m_{\text{GUT}}) > 0$, we find from (2.26)–(2.27) the following condition on N and the average messenger scale:

$$N \left(\log \frac{m_{\text{GUT}}}{\overline{\mathcal{M}}} \right) \lesssim 150 \tag{2.31}$$

In other words, we find the same condition as in OGM, but with the messenger scale given by $\overline{\mathcal{M}}$. At $\overline{\mathcal{M}} = 10^3, 10^5, 10^7, 10^9$ TeV, this condition allows for $N = 6, 8, 10, 15$ messengers, respectively.

3. Classification of models

Having deduced some general results about EOGM models, next we would like to identify three distinct categories of models and apply these results to each category.

3.1 Type I: theories with $\det m \neq 0$

In these theories, it is most convenient to use a bi-unitary transformation to go to a basis where m is diagonal. In this basis, the fields must come in pairs with R-charges $R(\phi_i) + R(\tilde{\phi}_i) = 2$. According to (2.3), this means

$$n = 0 \quad \text{and} \quad \det(\lambda X + m) = \det m \tag{3.1}$$

Note that (3.1) necessarily implies that $\det \lambda = 0$, otherwise the expansion of $\det(\lambda X + m)$ in powers of X would include the term $X^N \det \lambda$.¹⁰

Since these models have $\det m \neq 0$ and $\det \lambda = 0$, the messengers are all stable in a neighborhood of $X = 0$, but some of them can become tachyonic at large X . Thus, these models have a stable messenger sector only for

$$|X| < X_{\text{max}} \tag{3.2}$$

for some X_{max} which (if it is not infinite) depends on F and the other parameters of the model. Beyond this region of stability, the model will generally have runaway behavior, as seen in the examples of [33], and studied more generally in [40].

Because $n = 0$, these models are somewhat pathological phenomenologically: according to (2.5), the gaugino masses all vanish to leading order in F . In general, this leads to a large hierarchy between the gaugino and squark masses (even when higher order corrections in F/M_{mess}^2 are taken into account), which in turn exacerbates the fine-tuning problems of gauge mediation.

The type I category comprises the bulk (if not all) of the O’Raifeartaigh-based model-building literature. This includes some of the early attempts [4–8] at model building with

¹⁰Another, perhaps more direct way to prove these statements is the following: in the basis where m is diagonal, let us order the ϕ_i fields in increasing R-charge, $R(\phi_1) \leq R(\phi_2) \leq \dots \leq R(\phi_N)$. Then λ must be *strictly* upper triangular, since if $\lambda_{ij} \neq 0$, the selection rule $0 = R(\phi_i) + R(\tilde{\phi}_j) = R(\phi_i) - R(\phi_j) + 2$ requires $i < j$. This in turn implies all the statements above, namely that $\det \lambda = 0$, $\lambda X + m$ is an upper triangular matrix with only m on the diagonal, and the determinant of this matrix is independent of λ .

(simple variations on) the original O’Raifeartaigh model [41], as well as the more modern models of [21, 22] where many aspects of the $n = 0$ theories (including the vanishing of the gaugino masses) were worked out in detail. More recently, there have been many models [42–51] based on massive SQCD in the free-magnetic phase [26]; these also fall in the type I category, because the O’Raifeartaigh model of [26] is essentially a type I model. It is important to note that in many of the models listed above, the R-symmetry is not spontaneously broken by the interactions of the O’Raifeartaigh model itself. As a result, these models generally include additional interactions to break the R-symmetry either explicitly or spontaneously. Sometimes (e.g. when the R-symmetry is broken explicitly) these interactions can give rise to leading-order gaugino masses, thus avoiding the gaugino/squark mass hierarchy and its associated fine-tuning problems.

This is all we would like to say about the type I models, since these have been fairly well-explored in the literature. We would like to emphasize that the vanishing of the gaugino masses is not a feature of spontaneous R-symmetry breaking in general, but only of this particular, special category of models where $n = 0$. In the vast majority of EOGM models, $n \neq 0$ and the gaugino masses are nonzero at leading order in F , even with a spontaneously broken R-symmetry. We will focus on such models in the remainder of the paper.

3.2 Type II: theories with $\det \lambda \neq 0$

Here it is most convenient to diagonalize λ by a bi-unitary transformation. Then the fields must come in pairs with $R(\phi_i) + R(\tilde{\phi}_i) = 0$, and so

$$n = N \quad \text{and} \quad \det(\lambda X + m) = X^N \det \lambda \tag{3.3}$$

according to (2.3).¹¹ Note that the type II models include OGM as a special case ($m = 0$), as well as all continuous deformations of OGM consistent with the symmetries.

It is simple to sketch the messenger spectrum for the type II models, using the fact that $\det \lambda \neq 0$ and $\det m = 0$. At large X , $\det \lambda \neq 0$ implies that all the messengers have $\mathcal{O}(\lambda X)$ masses; thus

$$N_{\text{eff}}(X \rightarrow \infty) = N \tag{3.4}$$

i.e. the theory reduces to N -messenger ordinary gauge mediation at large X . As X approaches the origin, $\det m = 0$ means that some messengers have $\mathcal{O}(m)$ masses while others are much lighter, with masses that go to zero as some power of X . Eventually these light messengers must become tachyonic, and from this we learn that the type II models have a stable messenger spectrum for

$$|X| > X_{\text{min}} \tag{3.5}$$

for some X_{min} .

Note that these models do not suffer from the same problems as the type I models, since $n = N \neq 0$ means that the gaugino masses are nonzero at leading order in F/M_{mess}^2 . Thus,

¹¹As in the type I models, we can see these statements more directly by ordering the ϕ_i fields in decreasing R-charge, $R(\phi_1) \geq R(\phi_2) \geq \dots \geq R(\phi_N)$. Then m must be strictly upper triangular, since $2 = R(\phi_i) + R(\tilde{\phi}_j) = R(\phi_i) - R(\phi_j)$ requires $i < j$.

these models preserve the attractive feature of OGM whereby the gaugino and sfermion masses are generated at the same scale parametrically.

Another nice feature of this class of models has to do with unification. According to (3.3), $\det(\lambda X + m)$ is completely independent of m . Then according to (2.30), this means that $m_{2,3}$ can be split an arbitrary amount without any effect on unification. From the low-energy perspective, this would look like an amazing coincidence. For instance, if we take $\lambda_2 = \lambda_3 = \lambda$ and look in the regime $m_3 \ll \lambda X \ll m_2$, the doublet and triplet messenger spectra are completely different (following the sketch above). Nevertheless, the R-symmetry causes the messenger masses to be arranged in such a way that the gauge couplings still unify just as in the MSSM.

3.3 Type III: theories with $\det \lambda = \det m = 0$

These models have

$$0 < n < N \quad \text{and} \quad \det(\lambda X + m) = X^n G(m, \lambda) \tag{3.6}$$

according to (2.3). By dimensional analysis, $G(m, \lambda)$ must depend on both m and λ . Since $n \neq 0$, the gaugino masses are nonvanishing at leading order in F/M_{mess}^2 , as in the type II models.

Since $\det m = \det \lambda = 0$, the messenger spectrum in type III models combines features of the type I and type II models. In particular, there will be light messengers at both large and small X in these models. Thus these models generally have a stable messenger sector only for X in a window,

$$X_{\text{min}} < |X| < X_{\text{max}} \tag{3.7}$$

where again, X_{min} and X_{max} depend on the parameters of the model.

Type III models yield a variety of interesting theories which (to our knowledge) have never been discussed in the literature. One novel feature of these models is that it is fairly common to have $N_{\text{eff}} < 1$. For instance, we can see from the upper bound in (2.10) that this will happen at large X provided that n is sufficiently small (e.g. $n = 1$). This is a somewhat exotic scenario, and it allows us to achieve sfermion/gaugino mass ratios not ordinarily seen in gauge mediation. For instance, if we keep the sfermion masses fixed at some scale (say, to push the Higgs mass above the LEP bound), then taking $N_{\text{eff}} < 1$ makes the gauginos lighter than in OGM. Having extra-light gauginos in the spectrum (and the gluino in particular) could be interesting, as it could enhance sparticle production rates at the LHC relative to OGM scenarios. In section 4.2, we will analyze in detail the phenomenology of specific examples of type III models which have $N_{\text{eff}} < 1$.

4. Examples

4.1 Example 1: a family of type II models

In this section, we will consider some specific examples of EOGM models. These will serve to illustrate the general features discussed in the previous sections.

Let us start with a simple family of type II models:

$$\mathcal{M} = \lambda X + m = \begin{pmatrix} \lambda_1 X & m_1 & & & \\ & \ddots & \ddots & & \\ & & \ddots & \ddots & \\ & & & \ddots & m_{N-1} \\ & & & & \lambda_N X \end{pmatrix} \quad (4.1)$$

This family of models is the most general if we assign the following R-charges to the fields: $R(\phi_i) = -2i$, $R(\tilde{\phi}_i) = 2i$. The form of these models is motivated by the following considerations. In order to get the maximum effect from doublet/triplet splitting, we would like for the range of N_{eff} to be as large as possible. As discussed around (3.4), in the type II models $N_{\text{eff}}(X \rightarrow \infty) = N$, so to maximize the spread in N_{eff} we would like for $N_{\text{eff}}(X \rightarrow 0) = 1$. It turns out that (4.1) is the unique family of type II models which has $N_{\text{eff}}(X \rightarrow 0) = 1$.¹²

A simple choice for the λ 's and m 's that also satisfies the requirement of messenger parity is to make them maximally uniform:

$$\lambda_i = \lambda, \quad m_i = m \quad (4.2)$$

By a rephasing of all the fields, we can always take λ , m , X and F to be real in this family of examples. (As an aside, this shows that these particular EOGM models have no dangerous CP-violating phases.) In this case, messenger parity acts according to (2.16) with $U = \tilde{U}$ given by the permutation matrix $U_{ij} = \delta_{i, N-j+1}$, or equivalently $\phi_i \leftrightarrow \tilde{\phi}_{N-i+1}^*$.

The choice (4.2) leads to a nice simplification: by dimensional analysis, and because X always appears with a λ , $N_{\text{eff}}(X, m, \lambda)$ must be a function only of the dimensionless quantity

$$x = \frac{\lambda X}{m} \quad (4.3)$$

Shown in figure 2 are plots of $N_{\text{eff}}(x)$ for $N = 2, 3, 4, 5$. (As discussed in section 3.2, the type II models have a stable messenger sector only for $|X| > X_{\text{min}}$ for some X_{min} . In the following we will always be implicitly taking this bound into account.) We see that, by construction, $N_{\text{eff}}(x)$ interpolates between 1 and N .

Now we would like to include doublet/triplet splitting and see how it affects the phenomenology. Since unification depends only on λ_2, λ_3 (see section 3.2), we will set

$$\lambda_2 = \lambda_3 = 1 \quad (4.4)$$

for simplicity. Note that when all the λ 's are the same, the actual value of λ is irrelevant for the current discussion, since it always enters in the combinations λX and λF .

¹²Proof: from the lower bound in (2.9), we see that $N_{\text{eff}}(X \rightarrow 0) = 1$ requires $r_m = N - 1$. As discussed below (3.3), the matrix m must be strictly upper triangular in a basis where λ is diagonal and the R-charges are ordered $R(\phi_i) \geq R(\phi_{i+1})$. In order for m to have rank $N - 1$, it must have $m_{i, i+1} \neq 0$; then this fixes the R-charges of the fields uniquely (up to an overall phase rotation) to be $R(\phi_i) = -2i$, $R(\tilde{\phi}_i) = 2i$, which in turn forces all the other entries of m to be zero.

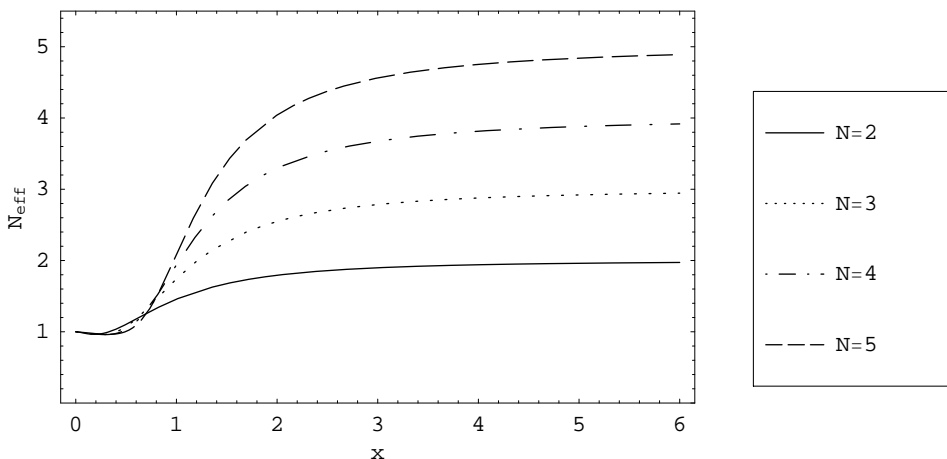


Figure 2: A plot of the effective messenger number $N_{\text{eff}}(x)$ vs. $x = \lambda X/m$.

We have generated MSSM spectra for a $N = 5$ model with $m_2 = 2X$ and $m_3 = \frac{1}{3}X$. This implies $N_{\text{eff},2} \approx 1$ and $N_{\text{eff},3} \approx 4.5$, as shown in figure 2. To fix the remaining parameters (X and F), we set $\Lambda_G = Nf/X = 200$ TeV and scanned over the mass of the lightest messenger. This choice of Λ_G leads to stop masses around $m_{\tilde{t}} \approx 1.5$ TeV and a Higgs mass around $m_{h^0} \approx 115$ GeV, which is consistent with the LEP bound, $m_{h^0} > 114.4$ GeV. Finally we have taken $\tan \beta = 20$ and $\mu > 0$ as a representative choice of these parameters. The spectra are shown plotted vs. the mass of the lightest messenger in figure 3. For comparison, the spectra for an OGM model with $N = 1$ and $N = 5$ messengers (and all the other parameters the same) are also shown in figure 3.

The spectra shown in figure 3 nicely illustrate some of the general points made in sections 2.3 and 2.4 about the effects of doublet/triplet splitting. For example, in the first row of figure 3 we see that in the EOGM model the squark and slepton masses are squashed in comparison to the $N = 1$ and $N = 5$ OGM models. In fact, since Λ_G was chosen to be the same in the three spectra, we see that the masses of colored (uncolored) sfermions are as in the $N = 5$ ($N = 1$) OGM model, in accord with the values of $N_{\text{eff},3}$ and $N_{\text{eff},2}$ respectively. By contrast, note that the values of M_1 , M_2 and $m_{\tilde{g}} \approx M_3$ are the same between all three models, since the R-symmetry implies that GUT relations (2.13) always hold for the gaugino soft masses.

The second row of figure 3 contains plots of μ and the masses of the lightest charginos, neutralinos, and stau; while the third row contains plots of the Higgsino components of the lightest neutralino and charginos. (For the standard definition of the Higgsino components see appendix C.) These plots further illustrate the consequences of doublet/triplet splitting, specifically the dramatic effects of “focussing” and small μ discussed in section 2.4. To see this, consider first the $N = 1$ and $N = 5$ OGM spectra in figure 3. These exhibit some well-known features of OGM: μ is always large, and either the bino ($N = 1$) or the stau ($N = 5$) is always the NLSP. Now contrast this with the EOGM spectrum shown in figure 3: because of the cancellation in the running of $m_{H_u}^2$ coming from $N_{\text{eff},3} \gg N_{\text{eff},2}$, this spectrum has

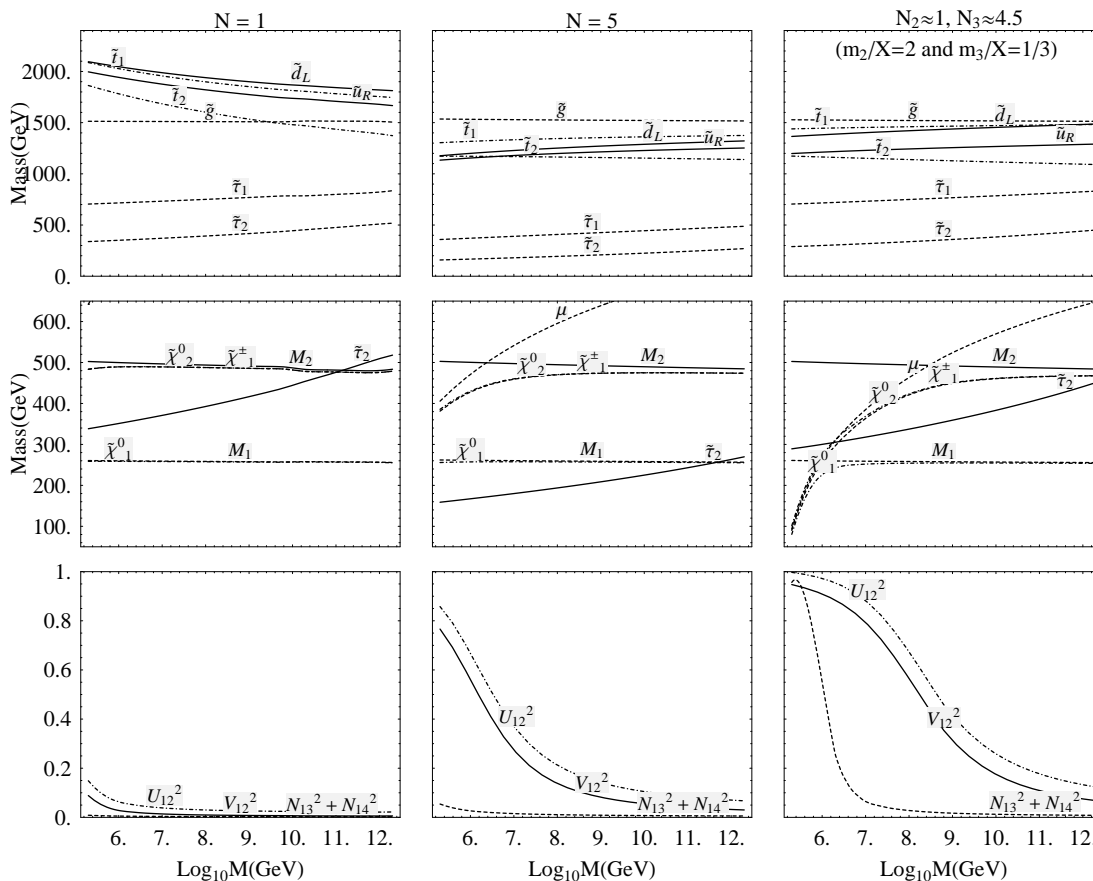


Figure 3: A plot of some of the MSSM soft parameters and sparticle masses at the scale $Q = m_Z$, as a function of the messenger scale M (which we take to be the mass of the lightest messenger). The left (middle) column is OGM with $N = 1$ ($N = 5$). The right column is a model of the form (4.1)(4.2) with $N = 5$, $m_2/X = 2$, and $m_3/X = 1/3$. In all cases, $\Lambda_G = 200$ TeV.

$\mu \lesssim M_1$ and a Higgsino NLSP at low messenger scales.

This point is further illustrated in figure 4, which contains contour plots of μ and the Higgsino component of the lightest neutralino vs. $m_2 > X$ and $m_3 < X$, for $N = 3, 4, 5, 6$. In these plots, we are holding fixed Λ_G and the mass of the lightest messenger; the parameters are chosen so that $m_{h^0} \approx 115$ GeV. For $N \geq 4$, we see that a sizable region of parameter space has $\mu < 200$ GeV as well as an NLSP neutralino that is more than 80% Higgsino.

Finally, let us see how gauge coupling unification works in this example, following the general discussion in section 2.6. Keep in mind that throughout this subsection, we have split the doublets and triplets in accordance with the determinant identity (2.3), so that unification proceeds with the same precision as in the MSSM. In figure 5, we show explicitly how the gauge couplings run in a model with $N = 3$, $m_2 = 2X$, $m_3 = \frac{1}{3}X$, $\Lambda_G = 200$ TeV, and lightest messenger mass $M_{\text{mess}} = 200$ TeV. For this model point, $N_{\text{eff},2} \approx 1$ and $N_{\text{eff},3} \approx 3$, so the lightest doublet and all three triplets contribute to the MSSM spectrum, while the two heavy doublets essentially serve only to preserve gauge

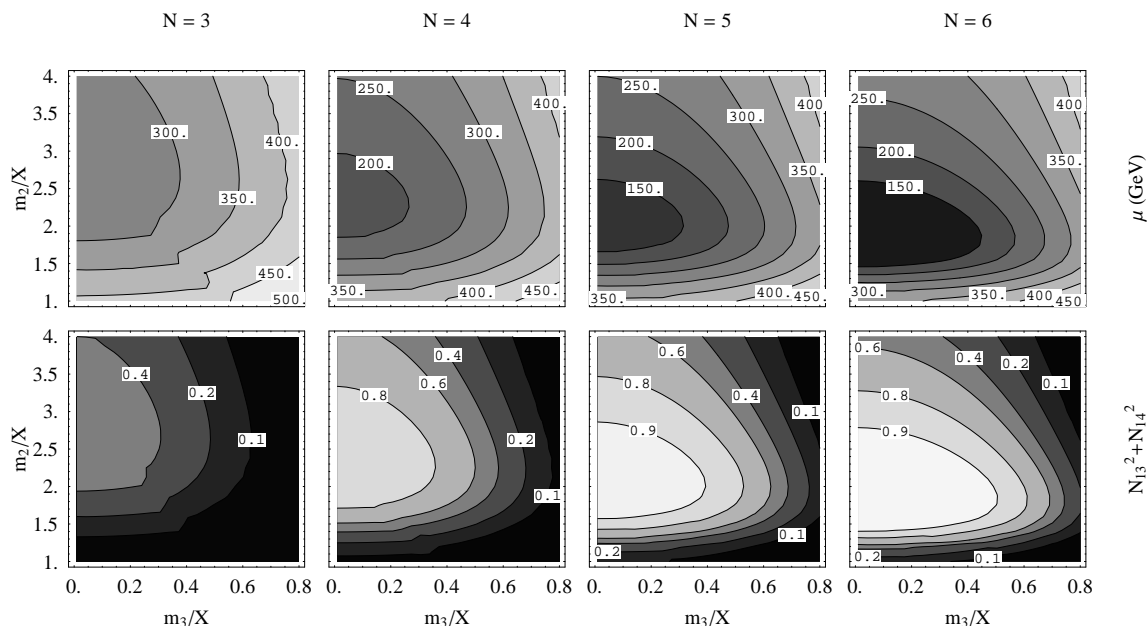


Figure 4: Contour plots of μ and $N_{13}^2 + N_{14}^2$ in $\{m_2/X, m_3/X\}$ space for $N = 3, 4, 5, 6$, $M_{\text{mess}} = 200 \text{ TeV}, 250 \text{ TeV}, 300 \text{ TeV}, 350 \text{ TeV}$, and $\Lambda_G = 200 \text{ TeV}$. This value of Λ_G corresponds to $M_1 = 260 \text{ GeV}$ and $M_2 = 520 \text{ GeV}$.

coupling unification. The solid lines in figure 5 indicate the running of the gauge couplings up to the GUT scale; in the magnified region around the GUT scale (inset), one can clearly see that the gauge couplings unify to a high degree of precision. Note that the running of the gauge couplings is very sensitive to the location of the messenger scales, so the R-symmetry is crucial for maintaining unification without tuning. This point is illustrated by the dashed lines in figure 5, which indicate the running of the gauge couplings for the same model point but with the two heavy doublet masses artificially raised by a factor of 10. In this case, unification is already off by a significant amount, as is clearly indicated in the inset. (Shown in the inset is also a band obtained by varying the input value of α_3 at M_z by 5%, which is meant to be a rough indication of the uncertainty on α_3 from unknown MSSM threshold corrections and experimental error.)

4.2 Example 2: a family of type III models

Next, let us consider a simple family of type III models which have $n = 1$ and consequently $N_{\text{eff}} < 1$ at large X . These models are constructed by combining a single OGM messenger with an $N - 1$ messenger type I model:

$$W = \lambda' X \phi_1 \tilde{\phi}_1 + m \sum_{i=2}^N \phi_i \tilde{\phi}_i + \lambda X \sum_{i=2}^{N-1} \phi_i \tilde{\phi}_{i+1} \quad (4.5)$$

This structure can easily be enforced by proper R-charge assignments. These models have $n = 1$ because the OGM messenger contributes $R(\phi_i) + R(\tilde{\phi}_i) = 0$ to the formula for n in (2.3), while the $N - 1$ type I messengers each contribute $R(\phi_i) + R(\tilde{\phi}_i) = 2$. For the

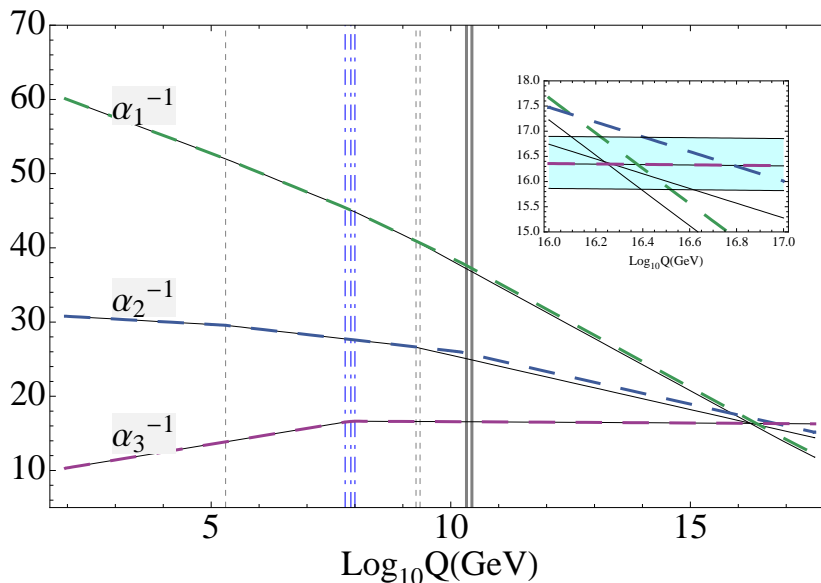


Figure 5: The gauge couplings vs. RG scale Q in an $N = 3$ EOGM model with $m_2 = 2X$, $m_3 = \frac{1}{3}X$. The vertical lines indicate the triplet (dot-dashed) and doublet (thin dashed) messenger masses. For comparison, the running of the gauge couplings is also shown (thick dashed) when the two heavy doublets are made 10 times heavier (thick solid). The inset is a magnification of the region $Q \sim m_{GUT}$. Shown in the inset is also a range for α_3 corresponding to varying $\alpha_3(M_z)$ by $\pm 5\%$.

choice of couplings in (4.5) (which can be taken to be real without loss of generality, as in example 1), they also have a messenger parity defined by (2.16) with $\tilde{U} = U$ and

$$U_{11} = 1, \quad U_{i1} = U_{1i} = 0, \quad U_{ij} = \delta_{i-2, N-j}, \quad (i \geq 2, j \geq 2). \quad (4.6)$$

Since the type I piece and the OGM piece are not directly coupled through the mass matrix, this messenger parity does not interchange OGM messengers with type I messengers. It is straightforward to verify (using e.g. (2.9)–(2.10)) that

$$N_{\text{eff}}(X \rightarrow 0) = 1, \quad N_{\text{eff}}(X \rightarrow \infty) = \frac{1}{N - 1 + (N - 2)^2} \quad (4.7)$$

Shown in figure 6 is N_{eff}^{-1} vs. X for these models with $N = 3, 4, 5$.

As discussed in section 3.3, the phenomenology of these models with $N_{\text{eff}} \ll 1$ can be quite interesting even without doublet/triplet splitting, because when $N_{\text{eff}} \ll 1$ the gauginos are lighter than usual. Shown in the first column of figure 7 is a sample spectrum with $N_{\text{eff}} \approx 1/3$, corresponding to an $N = 3$ model with $\lambda' = \lambda = 1$, $m = X/5$, $\Lambda_G = 90$ GeV, $\tan \beta = 20$ and $\mu > 0$. One sees from this that the gluino mass is around 700 GeV, even though the stops are still heavy at 1.5 TeV.

Lighter gauginos (and in particular the gluino) could mean an enhanced rate of sparticle production at the LHC, relative to more commonly studied OGM scenarios. Indeed, in collider studies of gauge mediation, it is often assumed that direct gluino production is highly suppressed relative to direct chargino and neutralino production, because the gluino

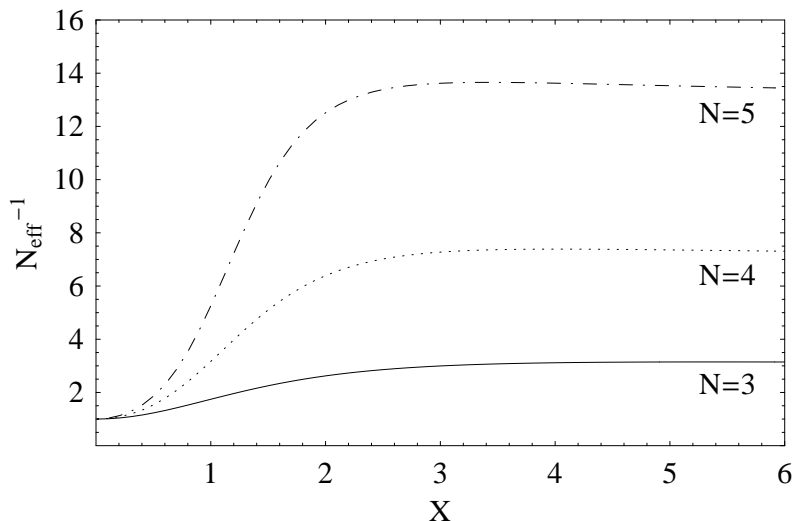


Figure 6: N_{eff}^{-1} vs. X for the model (4.5) for $N = 3, 4, 5$. For simplicity, the other parameters m , λ , λ' were all set to one.

mass is generally 1 TeV or more. However, we have seen here that in EOGM models it is possible to have $m_{\tilde{g}} \sim 700$ GeV. (The gluino mass could be lowered even further if we gave up the R-symmetry and the GUT relations.) Even between $m_{\tilde{g}} \sim 700$ GeV and $m_{\tilde{g}} \sim 1$ TeV, the difference in the direct gluino production rate at the LHC can be an order of magnitude or more, given the rapid fall off of the parton luminosity functions.

By including doublet/triplet splitting, it is possible to combine the features of type II and type III models discussed so far, i.e. to have a Higgsino NLSP *and* a light gluino. One reason such a scenario could be interesting is if it led to significantly enhanced Higgs production rates at the LHC. Note that maintaining unification is more complicated for type III models — there is not a clean separation in parameter space between the couplings that enter into $\det \mathcal{M}$ and couplings that do not. In this example, $\det \mathcal{M}$ depends on both m and λ' , but not λ . So if we want the same unification as in the MSSM, we can split only λ between the doublets and the triplets.

Shown in the second column of figure 7 is an example of a spectrum with both a Higgsino NLSP and light gluino. The model point corresponds to $N = 5$, $\lambda_2 = 1$, $\lambda_3 = 1/5$, and all the other parameters the same as in the previous example. Note that $N = 5$ is the minimum number of messengers required to obtain both a light gluino and a Higgsino NLSP. The reason is that one needs $N_{\text{eff},3}/N_{\text{eff},2} \gtrsim 3$ for small μ and Higgsino NLSPs (see figure 1), and $N_{\text{eff},3} \lesssim 1/3$ for light gluinos (as in the previous example). Together, this implies $N_{\text{eff},2} \lesssim 1/10$. As can be seen from (4.7), this is possible for $N \gtrsim 5$.

We should point out that it is rather more difficult to get both a Higgsino NLSP and a light gluino, compared to just one or the other. One reason is simply that if $m_{\tilde{g}} \sim 700$ GeV, then the GUT relations force $M_1 \sim 100$ GeV, which means there is only a very narrow window between $|\mu| = 0$ and $|\mu| \sim 100$ GeV where the NLSP has a significant Higgsino component. Another reason is that the combination of features requires some fine-tuning

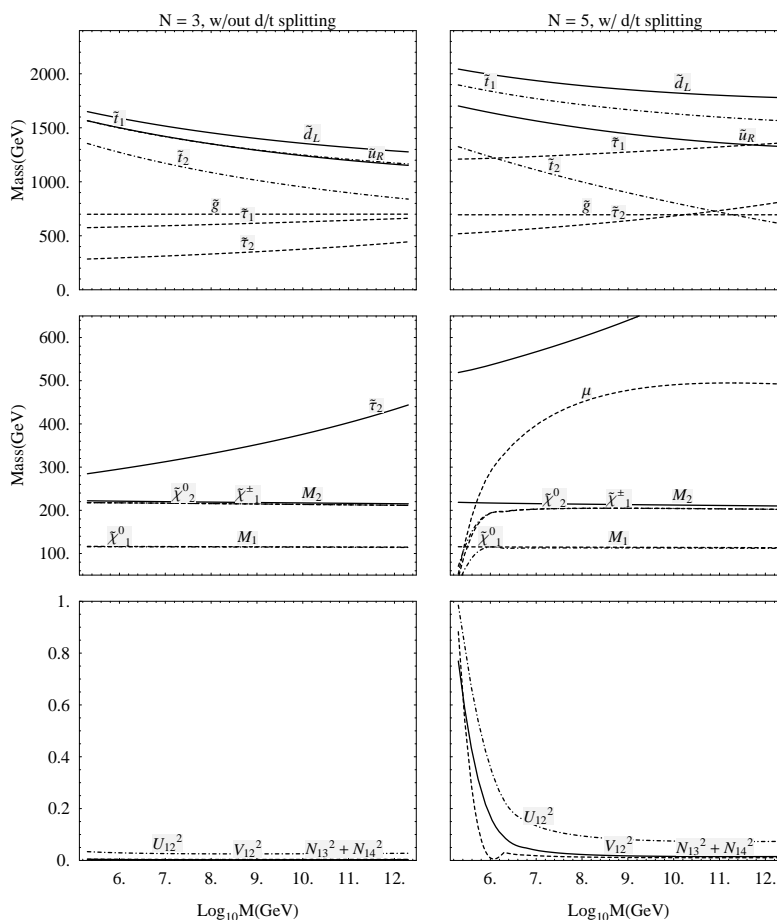


Figure 7: Example spectra with and without doublet/triplet splitting for the type III model (4.5). The left column has $N = 3$, $\lambda' = 1$, $\lambda = 1$, $m = X/5$, $\Lambda_G = 90 \text{ TeV}$, $\tan \beta = 20$ and $\mu > 0$. It shows that a 700 GeV gluino is possible in gauge mediation even keeping the stops heavy for the LEP Higgs mass bound. The right plot has the same parameters, except $N = 5$, $\lambda_2 = 1$, $\lambda_3 = 1/5$. Here we see that a light gluino and a Higgsino NLSP are simultaneously possible, at low messenger scales.

with respect to the superpotential parameters. To see this, note that in order to have both a Higgsino NLSP and a light gluino, we need $\lambda_2 X/m$ to take an asymptotic value for $N_{\text{eff},2} \approx 1/10$, but we need $\lambda_3 X/m$ to take an *intermediate* value for $N_{\text{eff},3} \approx 1/3$ (see figure 6). According to the discussion in section 2.5, this means that the cancellation in the running of the Higgs mass parameter (2.21) (which is controlled by $N_{\text{eff},3}/N_{\text{eff},2}$) depends sensitively on the superpotential parameters, unlike the case when X is asymptotic for both the doublets and the triplets.

5. Minimal completions of gauge mediation

5.1 Vacuum structure

So far, we have treated X as a spurion field whose vev and F-component are set by some

undetermined hidden sector. Thus, our approach up till this point has been analogous to most phenomenological studies of gauge mediation, where the details of the SUSY-breaking sector are not specified in order to be as model-independent as possible. Now, in the last section of the paper, we would like to go one step further and see what happens if we require $\langle X \rangle$ to be set by the renormalizable, perturbative dynamics of the EOGM model itself. We will see that these dynamics *can* result in a viable SUSY and R-symmetry breaking vacuum. Since the messengers play a vital role in the SUSY breaking, this means that the models studied in this paper can be viewed as minimal examples of direct gauge mediation.

Now let us describe our models in more detail. Given that we have imposed $R(X) = 2$ on our EOGM models, if we do not enlarge the matter content of the theory, then the only term we can add to the EOGM superpotential (2.1) that is renormalizable and consistent with the symmetries is

$$\delta W = FX \tag{5.1}$$

In other words, the minimal completions of our EOGM models are just generalized O’Raifeartaigh models:

$$W = \lambda_{ij} X \phi_i \tilde{\phi}_j + m_{ij} \phi_i \tilde{\phi}_j + FX \tag{5.2}$$

In general, because of the R-symmetry there is a SUSY-breaking pseudo-moduli space (i.e. a space of local minima of the tree-level scalar potential) at

$$\phi = \tilde{\phi} = 0, \quad X_{\min} \leq |X| \leq X_{\max} \tag{5.3}$$

for some X_{\min} and X_{\max} (which could be zero and infinity, respectively). In order for these models to be viable, the one-loop Coleman-Weinberg potential must have a local minimum on this pseudo-moduli space. Moreover, we need this minimum to occur at $\langle X \rangle \neq 0$, in order to break the R-symmetry and give the MSSM gauginos nonzero soft masses.

We should note that, even though we are referring to these models as generalized O’Raifeartaigh models, they generally have SUSY vacua or runaway behavior in addition to the pseudo-moduli space (5.3). (The R-symmetry, while necessary for SUSY-breaking, is not always sufficient [52].) Thus, the vacuum on the pseudo-moduli space (5.3) (if it exists) is only meta-stable, and it is important to make sure that it is sufficiently long-lived. Although we will not undertake a detailed analysis here, on general grounds we expect that the lifetime of the meta-stable vacuum is controlled by the small parameter λ . This is because, using the F-terms of (5.2) and the determinant identity (2.3), one can show that the SUSY vacuum or runaway direction in these models can only exist at $\phi \tilde{\phi} \sim 1/\lambda$ and $X = 0$ (or $X \rightarrow \infty$ in the case of runaway). So the parameter λ controls the separation in field space between the SUSY vacuum/runaway direction and the putative meta-stable vacuum at $\phi, \tilde{\phi} = 0, X \neq 0$. By making λ small, we should be able to make the latter parametrically long-lived.

5.2 More on R-symmetry breaking

It remains to determine whether, in a given model, there is a local minimum of the Coleman-Weinberg potential with $X \neq 0$. In [33], it was argued that this can only happen when

there exists a field with R-charge $R \neq 0, 2$. However, for technical reasons, the argument was limited to models with $\det m \neq 0$. Since we are interested in models with $\det m = 0$ in this paper, this argument cannot be directly applied. Nevertheless, the R-charge condition of [33] still seems to be true, even for models with $\det m = 0$. That is, regardless of whether m is degenerate or not, models where all the fields have $R = 0$ or 2 never seem to have R-symmetry breaking vacua at $X \neq 0$, while models with exotic R-charges do. In this subsection we would like to provide some heuristic arguments for why this should be the case.

To begin, recall that in this paper, we have been mostly interested in the regime $\sqrt{F} \ll m$, where the approximate formulas for the soft masses (2.5)–(2.6) make sense. In this regime, the Coleman-Weinberg potential simplifies – it reduces to derivatives of the effective Kähler potential (see e.g. appendix A of [26] for a detailed discussion of this),

$$V_{\text{CW}} \approx F^2 (K_{\text{eff}, XX^*})^{-1} \sim F^2 \partial_{XX^*}^2 \text{Tr } \mathcal{M}^\dagger \mathcal{M} \log \mathcal{M}^\dagger \mathcal{M} / \mu^2 \quad (5.4)$$

where for the sake of this heuristic discussion we are ignoring irrelevant constants and overall normalizations. Now, it is straightforward to apply this formula to our EOGM models and obtain a sketch of the CW potential at large and small X . At large X , we know on general grounds that

$$V_{\text{CW}} \sim F^2 \log X \quad (X \rightarrow \infty) \quad (5.5)$$

i.e. the potential grows monotonically like a logarithm. On the other hand, as we will now show, the behavior of (5.4) at small X (by which we mean $\sqrt{F} \ll X \ll m$) depends on the R-charge assignments of the fields.

First, let us consider a model where all the R-charges are 0 or 2. At $X \ll m$, the fields are either heavy, with $\mathcal{O}(m)$ masses, or light, with $\mathcal{O}(X)$ masses. Fields whose masses go like higher powers of X are forbidden by the R-charge assignments, as this would require a term $\sim X^m \tilde{\phi} \tilde{\phi}$ with $m > 1$ in the effective superpotential for the light field. Now, the contribution to the effective potential $V_{\text{CW}}^{(\text{heavy})}$ from the heavy messengers must be analytic in X, X^* ; therefore, the leading dependence on X in $V_{\text{CW}}^{(\text{heavy})}$ is $\mathcal{O}(|X|^2)$. On the other hand, it is straightforward to see from (5.4) that the light messengers contribute $\sim F^2 \log |X|$ to the potential. Thus the dominant contribution at small X to the potential comes from the light messengers, and moreover, we see that it is monotonically increasing. Given the behavior (5.5) at large X , the simplest possibility is that the entire potential grows monotonically with X and has no minimum at $X \neq 0$.

Next, let us consider a model with exotic R-charged fields. Here there can be ultra-light messengers with $\mathcal{O}(X^m)$ masses with $m \geq 2$. According to (5.4), these will contribute the following to the CW potential at small X ,

$$V_{\text{CW}}^{(\text{ultra-light})} \sim F^2 |X|^{2m-2} \log |X| \quad (5.6)$$

The crucial observation is that this contribution to the potential *decreases* at small X and eventually turns around at intermediate X . Therefore, the presence of a term like (5.6) in the potential can lead to a minimum away from the origin.

Note that the existence of such a minimum is still not guaranteed – the contributions from heavier messengers of the kind discussed above can overwhelm the effect of (5.6). We will see this happen, for instance, in some of the complete type II models to be discussed in the next subsection.

5.3 Type II completions

In this subsection and the next, we would like to study concrete examples of complete type II and type III models. We will see that the phenomenology of these models is more constrained than in the previous sections, since the vev of X can no longer be chosen arbitrarily.

Consider first the type II ($\det\lambda \neq 0$) models. As discussed in section 3.2, these models have a locally stable pseudo-moduli space at $\phi = \tilde{\phi} = 0$, as long as $|X| > X_{\min}$ for some X_{\min} . When $|X| < X_{\min}$, the potential either runs off to infinity or to a SUSY vacuum at $X = 0$, $\phi, \tilde{\phi} \neq 0$. As discussed above, as long as $\lambda \ll 1$, these features are well-separated from the pseudo-moduli space, and the SUSY-breaking meta-stable vacuum (if it exists) will be long-lived.

One nice feature of the type II completions is that as long as any $m_{ij} \neq 0$ (respecting an R-symmetry), there must be a field with $R \neq 0, 2$ in the theory.¹³ According to the discussion in the previous subsection, this means that the CW potential of all these models should have a SUSY and R-symmetry breaking minimum at $X \neq 0$, at least in some regime of parameters. Since the type II models with $m \neq 0$ comprise all the renormalizable, R-symmetric deformations of OGM, we have essentially shown that any such deformation of OGM — which by itself is an incomplete model — will lead to a complete model of gauge mediated SUSY breaking!

Now let us see how all this works in detail in a series of examples, presented in order of their complexity. The simplest example of a complete EOGM model is the $N = 2$ version of the models studied in section 4.1:

$$W = \lambda X(\phi_1 \tilde{\phi}_1 + \phi_2 \tilde{\phi}_2) + m\phi_1 \tilde{\phi}_2 + FX \tag{5.7}$$

Notice that $\delta W = m\phi_1 \tilde{\phi}_2$ is the only renormalizable deformation of $N = 2$ OGM consistent with *any* R-symmetry (up to permutations). In this model, the boson and fermion messenger masses can be calculated explicitly; substituting into the approximate CW potential (5.4), one finds (to leading order in F^2)

$$V_{\text{CW}} = \frac{5\lambda^2 F^2}{32\pi^2} V_2(x) \tag{5.8}$$

$$V_2(x) = -\frac{2}{4x^2 + 1} + 4 \log x + \frac{2x^2 + 1}{(4x^2 + 1)^{3/2}} \log \frac{2x^2 + 1 + \sqrt{4x^2 + 1}}{2x^2 + 1 - \sqrt{4x^2 + 1}}$$

where $x \equiv \lambda X/m$. The function $V_2(x)$ is plotted in figure 8; one finds by inspection that it is minimized at $x = 0.2494$.

¹³To see this, let us again go to a basis where λ_{ij} is diagonal. Then $R(\phi_i) + R(\tilde{\phi}_i) = 0$, and any $m_{ij} \neq 0$ implies $2 = R(\phi_i) + R(\tilde{\phi}_j)$, so either $\phi_i, \tilde{\phi}_i, \phi_j$ or $\tilde{\phi}_j$ must have $R \neq 0, 2$.

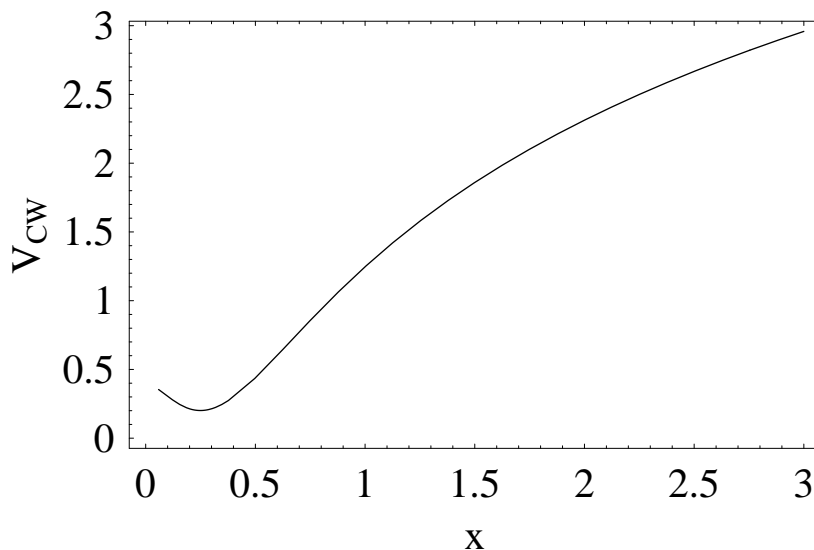


Figure 8: The CW potential for an $N = 2$ type II model (in arbitrary units), in the small F limit.

An analogous calculation of $V_N(x)$ for $N = 3, 4, 5$ reveals that x is minimized at $(0.38, 0.45, 0.5)$, respectively, and there is no minimum for $N \geq 6$.¹⁴ Therefore, the $N \leq 5$ models are extremely simple, complete models of direct gauge mediation.

Consider now the effect of doublet/triplet splitting in m_2, m_3 , keeping $\lambda_2 = \lambda_3 = \lambda$ for unification. Because of the structure of this model, the CW potential for X is straightforward to compute, and it is a simple sum of contributions from the doublet and triplet sectors:

$$V_{\text{CW}} = \frac{\lambda^2 F^2}{32\pi^2} (2V_N(x_2) + 3V_N(x_2\rho^{-1})) \quad (5.9)$$

where we have defined $x_2 = \lambda X/m_2$ and $\rho = m_3/m_2$. As described above, the first (second) term in (5.9) has a minimum around $x_2 \sim 1$ ($x_2 \sim \rho$). Thus when $\rho \gg 1$, the second term in the potential is very flat compared to the first, and V_{CW} is minimized around $x_2 \sim 1$. Meanwhile, for $\rho \ll 1$, the opposite is true, and the minimum of V_{CW} is at $x_2 \sim \rho$. The upshot is that the minimum of the potential always tracks the smaller of the two mass parameters, i.e. $\langle X \rangle \sim \min(m_2, m_3)$.

Notice that in these examples, the vacuum always ends up at $x < 1$ (or $x_2, x_3 < 1$ when there is doublet/triplet splitting). This seems to be a general feature of these models, and there is a simple intuitive reason for it. Namely, when $x \gtrsim 1$, the one-loop potential is basically that of N OGM messengers, i.e. it has no features and grows monotonically as a logarithm. Thus the minimum of the potential, if it exists, must occur at $x < 1$.

By construction (see the discussion below (4.1)), the examples considered so far have $N_{\text{eff}} \approx 1$ when $x < 1$. In order to build models with $N_{\text{eff}} > 1$, we need to take $r_m < N - 1$, i.e. there must be some number of OGM messengers. If for some reason we want to maximize $N_{\text{eff}}(x \rightarrow 0)$, then there should be as many OGM messengers as possible.

¹⁴This is an artifact of choosing $m_i = m, \lambda_i = \lambda$. Choosing these couplings to be different for the different messengers can lead to a CW potential with an R-symmetry breaking minimum.

Thus we are led to a model that is the sum of a two messenger type II model and $N - 2$ OGM messengers:

$$W = \lambda X(\phi_1 \tilde{\phi}_1 + \phi_2 \tilde{\phi}_2) + m\phi_1 \tilde{\phi}_2 + \lambda' X \sum_{i=3}^N \phi_i \tilde{\phi}_i + FX \quad (5.10)$$

This form of the superpotential could be enforced the R-symmetry, or by a $\mathbb{Z}_2 \times \mathbb{Z}_2$ symmetry that acts separately on the OGM and the type II messengers. Note that this model has a messenger parity symmetry which is simply the product of the separate messenger parities of the type II and the $N - 2$ OGM models. In this model, the lower bound in (2.9) implies that

$$N_{\text{eff}} \gtrsim \frac{N^2}{N+2} \quad (5.11)$$

when $x \lesssim 1$. So a minimum of the CW potential, if it exists, is guaranteed to have $N_{\text{eff}} > 1$. In these models, the CW potential takes the form (again at small F)

$$V_{\text{CW}} = \frac{5F^2}{32\pi^2} (\lambda^2 V_2(x) + 2(N-2)\lambda'^2 \log x) \quad (5.12)$$

so the condition for the existence of a minimum is

$$\lambda' \ll \lambda \quad (5.13)$$

Otherwise, the contribution from the type II messengers (which has a minimum at $x \approx 0.25$) will be overwhelmed by the monotonically growing contribution from the OGM messengers.

Finally, in order for doublet/triplet splitting to lead to $N_{\text{eff},3} \neq N_{\text{eff},2}$, we need to construct a model that interpolates between (5.10) and the higher N generalizations of (5.7), while remaining consistent with messenger parity. We can achieve this with the following model:

$$W = \lambda X(\phi_{-1} \tilde{\phi}_{-1} + \phi_1 \tilde{\phi}_1) + m\phi_{-1} \tilde{\phi}_1 + \lambda' X \sum_{i=2}^{\frac{N}{2}} (\phi_i \tilde{\phi}_i + \phi_{-i} \tilde{\phi}_{-i}) \quad (5.14)$$

$$+ \delta m \sum_{i=1}^{\frac{N}{2}-1} (\phi_i \tilde{\phi}_{i+1} + \phi_{-i} \tilde{\phi}_{-i-1}) + FX$$

where to maintain messenger parity, we have coupled the two-messenger type II model in a symmetric way to two $(N-2)/2$ -messenger models. When $\delta m \rightarrow 0$, this model reduces to (5.10), and when $\delta m \rightarrow m$ this model becomes the higher N generalization of (5.7) (albeit with split λ, λ'). Note that this particular interpolating model only works if the total number of messengers is even. By having different δm for the doublets and triplets, we can make $N_{\text{eff},3} \gg N_{\text{eff},2}$ and obtain all the exotic phenomenology (Higgsino NLSP, small μ , etc.) discussed in the previous sections, all within the context of a complete model. To illustrate this, we have generated in figure 9 contour plots of μ and the Higgsino component of the lightest neutralino, for models of the form (5.14) with $N = 4, 6$; $\lambda' = \lambda/10$ (to satisfy (5.13)); and $\delta m_2 = m_2/10$, $\delta m_3 = 0$ so that $N_{\text{eff},3}$ is given by (5.11) and

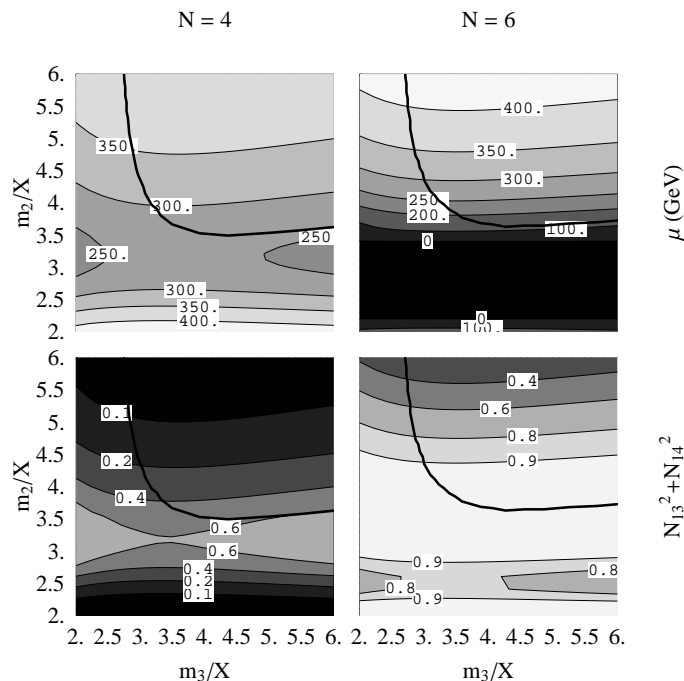


Figure 9: Contour plots for μ in $\{m_2/X, m_3/X\}$ space for $N = 4, 6$, $M_{\text{mess}} = 200 \text{ TeV}, 400 \text{ TeV}$, and $\Lambda_G = 200 \text{ TeV}, 300 \text{ TeV}$. Here $\lambda' = \lambda/10$, $\delta m_2 = m_2/10$, and $\delta m_3 = 0$. The solid line denotes the values of $\langle X \rangle$ corresponding to the complete model.

$N_{\text{eff},2} \approx 1$. These contour plots are scanned over m_2/X and m_3/X , again treating X as a free parameter. The special case where X is determined by the Coleman-Weinberg potential is indicated by the solid line in figure 9.

Let us conclude this subsection with a short summary of our results so far. First, we have argued that the type II EOGM models lead naturally to extremely compact, complete models of direct gauge-mediated SUSY-breaking. We have also seen that the simplest models have $N_{\text{eff}} \approx 1$ and are largely insensitive to doublet/triplet splitting. So in a sense, these features could be viewed as generic predictions of these minimal models. Finally, we constructed complete models with $N_{\text{eff}} > 1$ and $N_{\text{eff},3} \gg N_{\text{eff},2}$, using the more complicated setups (5.10) and (5.14). The latter models are rather contrived,¹⁵ and they are only intended to be existence proofs, showing that the exotic phenomenology discussed in previous sections is possible within the space of these minimal completions of gauge mediation.

5.4 Type III completions

We would also like to explore completions of type III models. As we have discussed, the most interesting effects of type III models occur when $n = 1$, since this allows for

¹⁵In particular, why should $\delta m_2 \neq 0$ while $\delta m_3 = 0$? Note that this question is similar to the standard Higgs doublet/triplet splitting problem, with the role of doublets and triplets reversed. There have been many ideas on how to solve the Higgs doublet/triplet splitting problem (for a nice overview, see [53]), and perhaps some of these ideas can be applied here.

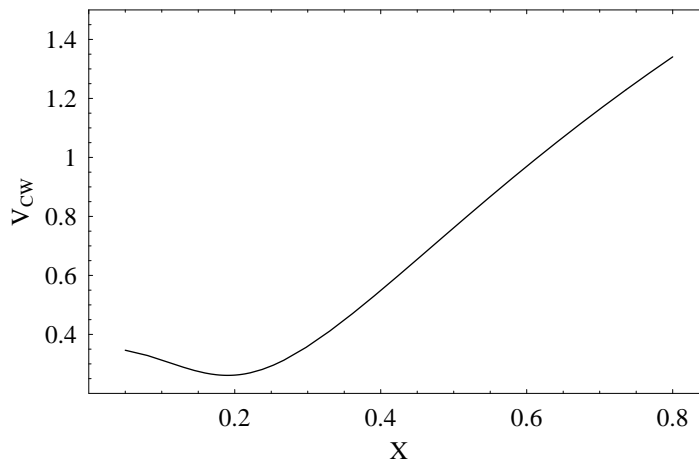


Figure 10: A plot of the CW potential for the complete $N = 4$ type III model discussed in the text, with $\lambda' = 0.15$, $\lambda = 1$, $m = 0.1M$, and $F = 10^{-4}M^2$.

the smallest possible N_{eff} . Thus, we will focus on completions of $n = 1$ models in this subsection. One can show that theories with $n = 1$ always contain a supersymmetric vacuum at $X = 0$, $\phi, \tilde{\phi} \neq 0$. As in the previous subsections, we will always assume that this SUSY vacuum is sufficiently far away from the SUSY-breaking pseudo-moduli space, so that the meta-stable vacuum (if it exists) is long-lived.

It is straightforward to take the models (4.5) discussed in section 4.2 and use them to build complete $n = 1$ models with exotic phenomenology. Recall that these models were combinations of type I models and OGM messengers. In this section, we will focus on a model of the form (4.5) with $N = 4$ messengers,

$$\lambda X + m = \begin{pmatrix} \lambda' X & 0 & 0 & 0 \\ 0 & m & \lambda X & 0 \\ 0 & 0 & M & \lambda X \\ 0 & 0 & 0 & m \end{pmatrix} \quad (5.15)$$

which respects the same messenger parity of (4.5) even with $m \neq M$. The CW potential for this model splits into a potential for the OGM messenger and a potential for the type I model; at small F this is given by

$$V_{\text{CW}} = \frac{5\lambda'^2 F^2}{16\pi^2} \log X + V_{\text{CW}}^{(\text{typeI})} \quad (5.16)$$

The type I model is precisely the one discussed in [33]; thus, we know that it has a minimum at $X \neq 0$ when $m \ll M$. In order for the OGM messenger not to destabilize this vacuum, we must also require $\lambda' \ll \lambda$. An example of a potential with an R-symmetry breaking minimum is shown in figure 10; here we have chosen $\lambda' = 0.15$, $\lambda = 1$, $m = 0.1M$, and $F = 10^{-4}M^2$.

With doublet/triplet splitting, it is possible to obtain complete models whose spectra contain light gluinos, as well as small μ and Higgsino NLSPs. As discussed in section 4.2, we can split λ (but not λ', m or M) between the doublets and triplets without affecting

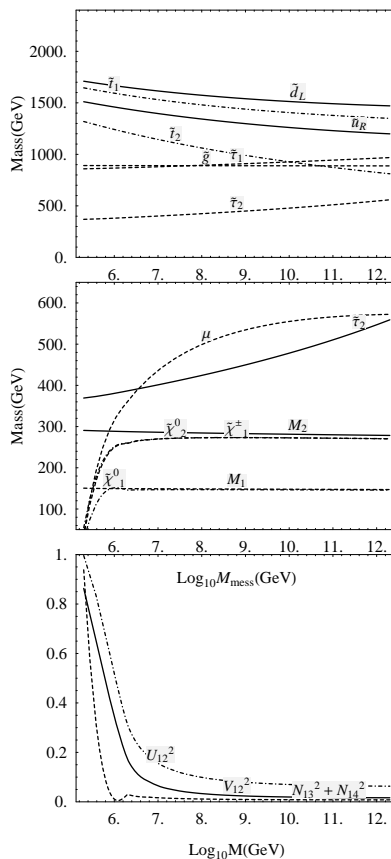


Figure 11: A sample spectrum for the complete type III model with doublet/triplet splitting discussed in the text. The parameters are as in the previous figure, but with $\lambda_2 = 1.75$ and $\Lambda_G = 115$ TeV.

unification. An example spectrum with split λ 's is shown in figure 11; here the scale is set with $\Lambda_G = 115$ TeV, and the parameters are the same as those in figure 10, except $\lambda_2 = 1.75$. Note that for this choice of parameters, $N_{\text{eff},3} = 0.6$ and $N_{\text{eff},2} = 0.2$. Figure 11 shows that it is possible to get gluino masses lighter than 1 TeV, as well as Higgsino NLSPs at low messenger scales, in a complete type III model.

Acknowledgments

First and foremost, we would like to thank M. Dine for enlightening discussions, correspondence, and comments on the draft. We would also like to acknowledge N. Arkani-Hamed, P. Meade, R. Rattazzi for useful discussions and N. Seiberg for useful discussions and comments on the draft. The research of LF is supported in part by NSF Graduate Research Fellowship. The research of CC is supported in part by DOE grant DE-FG02-91ER40654. The research of DS is supported in part by DOE grants DE-FG02-91ER40654 and DE-

FG02-90ER40542. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.

A. The messenger mass matrix

A.1 Determinant identity

Here we will prove that the R-symmetry selection rules (2.2) imply the identity (2.3):

$$\det \mathcal{M} = X^n G(m, \lambda), \quad \text{where } n = \frac{1}{R(X)} \sum_{i=1}^N \left(2 - R(\phi_i) - R(\tilde{\phi}_j) \right), \quad (\text{A.1})$$

To begin, recall the definition of the determinant

$$\det \mathcal{M} = \sum_{\sigma \in S_N} \text{sgn}(\sigma) \mathcal{M}_{1,\sigma(1)} \mathcal{M}_{2,\sigma(2)} \cdots \mathcal{M}_{N,\sigma(N)} \quad (\text{A.2})$$

where S_N is the degree N permutation group. Now consider any nonvanishing term in the sum (A.2), and define

$$T_{i,\sigma} \equiv R(\phi_i) + R(\tilde{\phi}_{\sigma(i)}) \quad (\text{A.3})$$

From the R-symmetry, $\mathcal{M}_{i,\sigma(i)}$ vanishes unless $T_{i,\sigma} = 2 - R(X)$ or 2. Furthermore, if $T_{i,\sigma} = 2 - R(X)$ ($T_{i,\sigma} = 2$) then $\mathcal{M}_{i,\sigma(i)}$ is proportional to X (a constant). Therefore, the nonvanishing term in question is a monomial in X , of degree

$$n = \sum_{i=1}^N \frac{(2 - T_{i,\sigma})}{R(X)} = \frac{1}{R(X)} \sum_{i=1}^N \left(2 - R(\phi_i) - R(\tilde{\phi}_{\sigma(i)}) \right) = \frac{1}{R(X)} \sum_{i=1}^N \left(2 - R(\phi_i) - R(\tilde{\phi}_i) \right) \quad (\text{A.4})$$

Note that the dependence on the permutation σ has dropped out in the last equation because of the sum over N . Therefore, every non-vanishing contribution to the determinant is proportional to X^n with the same power n , and this completes the proof of (2.3).

A.2 Messenger spectrum and the asymptotic behavior of N_{eff}

We would like to get some idea of how N_{eff} depends on the parameters of the model. But first, we need to get a rough picture of the messenger spectrum. For this purpose, the notation introduced in (2.11) will be useful:

$$r_\lambda \equiv \text{rank } \lambda, \quad r_m \equiv \text{rank } m \quad (\text{A.5})$$

Note that $r_\lambda + r_m \geq N$ necessarily, otherwise $\lambda X + m$ would be degenerate.

At large X , r_λ messengers have $\mathcal{O}(X)$ masses. The remaining $N - r_\lambda$ messenger masses must scale with a smaller power of X ,

$$\mathcal{M}_i \sim \frac{m^{n_i+1}}{X^{n_i}} \quad (\text{A.6})$$

where $n_i \geq 0$ and, according to the determinant identity (2.3),

$$\sum_{i=1}^{N-r_\lambda} n_i = r_\lambda - n \quad (\text{A.7})$$

On the other hand, at small X , r_m of the messengers have $\mathcal{O}(m)$ masses. According to (2.3), the remaining $N - r_m$ messengers have

$$\mathcal{M}_i \sim \frac{X^{n'_i+1}}{m^{n'_i}} \quad (\text{A.8})$$

masses, with $n'_i \geq 0$ and

$$\sum_{i=1}^{N-r_m} n'_i = n - (N - r_m) \quad (\text{A.9})$$

(By successively integrating out messengers, it is straightforward to prove that all the n_i and n'_i must be integers.) Together, these identities imply

$$N - r_m \leq n \leq r_\lambda \quad (\text{A.10})$$

As a check, note that this inequality is consistent with the inequality $r_\lambda + r_m \geq N$ deduced above.

Based on this picture of the messenger spectrum, it is trivial to derive using (2.7) the asymptotic behavior of N_{eff} as $X \rightarrow 0$ and as $X \rightarrow \infty$:

$$N_{\text{eff}}(X \rightarrow 0) = \frac{n^2}{\sum_{i=1}^{N-r_m} (n'_i + 1)^2}, \quad N_{\text{eff}}(X \rightarrow \infty) = \frac{n^2}{r_\lambda + \sum_{i=1}^{N-r_\lambda} n_i^2} \quad (\text{A.11})$$

Note that in both the $X \rightarrow 0$ and $X \rightarrow \infty$ limits, N_{eff} is invariant under any continuous deformations of m and λ which preserve the R-charge assignments.

Finally, combining (A.7), (A.9) and (A.11), together with the classic RMS-AM inequality $\langle x^2 \rangle \geq \langle x \rangle^2$, it is straightforward to show that the asymptotic values of N_{eff} satisfy the bounds (2.9)–(2.10) quoted in the text.

B. Renormalization methodology

In this section, we describe how the low-energy spectra exhibited in sections 3-5 were computed, in particular the threshold corrections and β functions that were used to run the soft masses from the messenger scale down to the weak scale. The formulae for all the corrections we used are presented in [54] and [55]. Typically, 10% accuracy in the low-energy soft parameters would be sufficient for the level of phenomenological detail that concerns this paper; however, we required much better than this since one of the most significant effects in the low-energy spectrum was a large cancellation in the running of $m_{H_u}^2$ between the messenger and the weak scale. We have therefore included radiative contributions in [54] or [55] that correct $m_{H_u}^2$ at the percent level.

We begin by detailing the renormalization group equation (RGE) effects. The most straightforward of these are the two-loop β functions, which we have only included for gaugino masses, α_3 , y_t , and $m_{H_u}^2$ itself. All other β functions are evaluated at 1-loop.

In addition, there are new RGE effects from the messengers. Below the scale of the scale M_{mess} of the lightest messenger, the RGE's are the familiar ones of the MSSM and have been worked out explicitly many places. However, above M_{mess} , the RGE's are modified. In the class of models considered in this article, there typically appear messengers at several different mass scales. Above the scale of the heaviest messenger, the lagrangian is supersymmetric, and all soft SUSY breaking terms vanish. In simple gauge mediation models where all of the messengers have the same mass, the soft terms are generated only in the low energy theory (MSSM) where the messengers have been integrated out, and so the messengers do not contribute to the running. With multiple messenger thresholds, however, the soft SUSY breaking terms begin to run as soon as the heaviest messenger is integrated out. In between messenger thresholds, the RGE's are those of the MSSM plus a contribution from the messengers. The contribution to the running of a scalar (mass)²'s for a general (softly broken) supersymmetric theory has been worked out in [54]. Contributions from the MSSM enter already at one-loop and so are naively much larger than the contribution from the messengers. However, by dimensional analysis they are proportional to MSSM (mass)², which are themselves suppressed by $(\alpha/4\pi)^2$, and therefore effectively give only a three-loop contribution to the running. Thus, the leading contribution is at order $\mathcal{O}(\alpha^2)$ and comes from the messenger sector (eq. (2.20) in [54]):

$$\frac{dm_{\tilde{f}}^2}{d \log Q} \approx \sum_{a=1}^3 8 \frac{g_a^4}{(4\pi)^4} C_{\tilde{f}}^a \text{Str}(S(r)\mathcal{M}^2) \tag{B.1}$$

Here, \mathcal{M}^2 denotes the messenger mass matrix (bosons and fermions), and $\text{tr}(t_r^A t_r^B) = S(r)\delta^{AB}$ defines the Dynkin index of the representation r .¹⁶ (B.1) has no effect above the scale of the heaviest messenger, where the supertrace theorem clearly holds, $\text{Str} \mathcal{M}^2 = 0$; or below the scale of the lightest messenger, where the supertrace is empty. In between the heaviest and the lightest messenger scale, however, (B.1) has an effect, and it is typically quite significant. We therefore include this contribution to the MSSM β functions in between messenger scales. Since we include all running between messenger scales, the threshold corrections to sfermion and gaugino masses from each messenger is evaluated with the renormalization scale equal to the messenger's own mass.

There are also many threshold corrections from the MSSM that are important to include. In particular, $m_{H_u}^2(m_{\tilde{t}})$ is extremely sensitive to the top Yukawa coupling and, to a lesser extent, α_3 . MSSM threshold corrections at the weak scale can change y_t (α_3) by around 10% (20%), which in turn corrects $m_{H_u}^2(m_{\tilde{t}})$ by around 50% when there is no focussing, and over 100% when there is. We include corrections to α_3 from stop and gluino

¹⁶More precisely, $S_a(r)$ is the Dynkin index for a single messenger ϕ_r for the gauge group G_i ; when the doublets and triplets are split, $S_1(2) = \frac{1}{2}(\frac{3}{5})$ and $S_1(3) = \frac{1}{2}(\frac{2}{5})$ are the dynkin indices for the U(1) gauge group, for a complete doublet field and triplet field respectively.

		y_t	y_b	y_τ	α_3	α_2	α_1	$m_{H_u}^2$	m_{h^0}
$N = 5$ EOGM	$\beta^{(2)}$	-0.027%	0.82%	-0.26%	-0.39%	0.056%	0.052%	-46.%	-0.033%
	Δ_{MSSM}	8.4%	9.1%	-0.28%	17.%	2.7%	1.9%	270 %	0.37%
	β_{mess}	0.020%	0.23%	-0.060%	0.0026%	0.018%	0.016%	-14.%	-0.041%
$N = 5$ OGM	$\beta^{(2)}$	-0.044%	0.39%	-0.060%	-0.38%	0.015%	0.011%	-14.41%	-0.043%
	Δ_{MSSM}	8.20%	11.38%	-1.71%	16.82%	2.99%	1.60%	83.77%	0.65%
$N = 1$ OGM	$\beta^{(2)}$	-0.036%	0.18%	0.0064%	-0.37%	0.018%	0.0032%	-8.18%	-0.029%
	Δ_{MSSM}	8.83%	11.23%	-0.78%	18.37%	3.18%	2.20%	88.31%	-0.069%

Table 1: The size of the correction to running parameters at 1 TeV. $\beta^{(2)}$ indicates the correction from 2-loop running, Δ_{MSSM} indicates the threshold corrections from the MSSM, and β_{mess} indicates the correction from running between messengers.

loops:

$$\Delta\alpha_3 = \frac{\alpha_3(M_Z)}{2\pi} \left[\frac{1}{2} - \frac{2}{3} \ln \left(\frac{m_t}{M_Z} \right) - 2 \ln \left(\frac{m_{\tilde{g}}}{M_Z} \right) - \frac{1}{6} \sum_{\tilde{q}} \sum_{i=1}^2 \ln \left(\frac{m_{\tilde{q}_i}}{M_Z} \right) \right] \quad (\text{B.2})$$

The running top Yukawa gets threshold corrections from squarks and gluino loops, as well as from neutralinos, charginos, and Higgses. We include all threshold corrections at 1-loop to y_t (eqs. D.16 and D.18 in [55]).

In addition, we include less significant threshold corrections to the standard model quarks and gauge couplings. In particular, we include all 1-loop threshold corrections to α_1 and α_2 ; these can be important, because they feed into the definition of the running Higgs vev $v^2 = 2m_Z^2/4\pi(\alpha_1 + \alpha_2)$, which in turn feeds into the definition of the running top mass.

To determine the low-energy MSSM spectrum, we employ an iterative procedure (as in standard programs, such as SOFTSUSY 2.0 [56]) whereby an initial guess at the messenger scale is RG evolved down to the weak scale, the MSSM threshold corrections are computed, these are used to update the high-scale boundary conditions, and this process is repeated until it converges to within a 2% change in μ^2 . Typically, this occurs within a few iterations.

We have checked that in the case of OGM with $N = 1$ or $N = 5$ messengers, our codes matches the results of SOFTSUSY 2.0 around the electroweak scale to 3% or better for all parameters and to 1% or better for $m_{H_u}^2$.

Table 1 summarizes the effect of these corrections on the soft masses for an EOGM point with small μ . Specifically, we have taken the type II model of section 4.1 with $m_2 = 3, m_3 = 1/2, \lambda = 1, X = 1, M_{\text{mess}} = 200$ TeV and $\Lambda_G = 160$ TeV. For comparison, the size of the effects are shown for $N = 1, 5$ OGM models with the same M_{mess} and Λ_G . The number in the table is $\frac{X_{\text{approx}} - X}{X}$ where X denotes the parameter with all corrections and X_{approx} omits the indicated correction. All the running parameters are evaluated at 1 TeV. $\beta^{(2)}$ denotes the two-loop running, and Δ_{MSSM} denotes threshold corrections from the MSSM. For EOGM, we also show the effect (β_{mess}) of running between messenger masses.

C. Phenomenology of a Higgsino-like NLSP

C.1 Masses and mixings

Because EOGM allows for a small μ parameter, the Higgsinos can be lighter than the gauginos, and so the NSLP can be Higgsino-like. To see this, recall the mass matrix for the neutralinos and charginos:

$$\begin{aligned}
 M_{\tilde{N}} &= \begin{pmatrix} M_1 & 0 & -c_\beta s_W m_Z & s_\beta s_W m_Z \\ 0 & M_2 & c_\beta c_W m_Z & -s_\beta c_W m_Z \\ -c_\beta s_W m_Z & c_\beta c_W m_Z & 0 & -\mu \\ s_\beta s_W m_Z & -s_\beta c_W m_Z & -\mu & 0 \end{pmatrix} \\
 M_{\tilde{C}} &= \begin{pmatrix} 0 & \mathbf{X}^T \\ \mathbf{X} & 0 \end{pmatrix} \quad \mathbf{X} = \begin{pmatrix} M_2 & \sqrt{2} s_\beta m_W \sqrt{2} c_\beta m_W \mu \end{pmatrix}
 \end{aligned} \tag{C.1}$$

In the limit $\mu \sim m_Z \ll M_1, M_2$, the masses are given by:

$$\begin{aligned}
 m_{\tilde{N}_{\pm}}^2 &= \mu^2 \pm \frac{\mu m_Z^2 (M_1 c_W^2 + M_2 s_W^2) (1 \mp \sin 2\beta)}{M_1 M_2} + \dots \\
 m_{\tilde{N}_3}^2 &= M_1^2 + 2m_Z^2 s_W^2 + \frac{2\mu m_Z^2 s_W^2 \sin 2\beta}{M_1} + \dots \\
 m_{\tilde{N}_4}^2 &= M_2^2 + 2m_Z^2 c_W^2 + \frac{2\mu m_Z^2 c_W^2 \sin 2\beta}{M_2} + \dots
 \end{aligned} \tag{C.2}$$

and

$$\begin{aligned}
 m_{\tilde{C}_1}^2 &= \mu^2 - \frac{2\mu m_W^2 \sin 2\beta}{M_2} + \dots \\
 m_{\tilde{C}_2}^2 &= M_2^2 + 2m_W^2 + \frac{2\mu m_W^2 \sin 2\beta}{M_2} + \dots
 \end{aligned} \tag{C.3}$$

The mass matrices (C.1) are diagonalized by a bi-unitary transformation $\tilde{N}_i = N_{ij} \psi_j^0$, $\tilde{C}_i^+ = V_{ij} \psi_j^+$, $\tilde{C}_i^- = U_{ij} \psi_j^-$. In the small μ limit, the bino, wino and Higgsino components of the lightest neutralinos and charginos are given by the formulae:

$$\begin{aligned}
 \{N_{\pm 1}^2, N_{\pm 2}^2, N_{\pm 3}^2 + N_{\pm 4}^2\} &= \\
 &\left\{ \frac{m_Z^2 s_W^2 (1 \mp \sin 2\beta)}{2M_1^2}, \frac{m_Z^2 c_W^2 (1 \mp \sin 2\beta)}{2M_2^2}, 1 - \frac{1}{2} m_Z^2 \frac{M_1^2 c_W^2 + M_2^2 s_W^2}{M_1^2 M_2^2} (1 \mp \sin 2\beta) \right\} + \dots \\
 \{U_{11}^2, U_{12}^2\} &= \left\{ \frac{2m_W^2 c_\beta^2}{M_2^2}, 1 - \frac{2m_W^2 c_\beta^2}{M_2^2} \right\} \\
 \{V_{11}^2, V_{12}^2\} &= \left\{ \frac{2m_W^2 s_\beta^2}{M_2^2}, 1 - \frac{2m_W^2 s_\beta^2}{M_2^2} \right\}
 \end{aligned} \tag{C.4}$$

Thus, in the small μ limit, the lightest neutralinos and charginos are almost completely Higgsino.

C.2 Decay rates

OGM has the well-known collider signature $\gamma\gamma + \cancel{E}$ from promptly decaying bins. The rates for this are typically enormous (there will be thousands of such events at the LHC after only 100 pb⁻¹ of data), and the SM backgrounds are virtually non-existent [57–60]. As such, $\gamma\gamma + \cancel{E}_T$ offers an excellent channel for early discovery of gauge mediation at the LHC.

In EOGM, a Higgsino NLSP can lead to a completely different collider signature. Because the Higgsino is the superpartner of the Higgs, which in turn mixes with the longitudinal mode of the Z, the branching ratio of the NLSP to these modes is larger than in OGM. The relative decay rates of NLSP to Goldstino + boson are given by

$$\begin{aligned} \frac{\Gamma(\chi_1^0 \rightarrow \tilde{G}Z^0)}{\Gamma(\chi_1^0 \rightarrow \tilde{G}\gamma)} &= \frac{\kappa_Z}{\kappa_\gamma} \left(1 - \frac{m_Z^2}{m_{\chi_1^0}^2}\right)^4 \\ \frac{\Gamma(\chi_1^0 \rightarrow \tilde{G}h^0)}{\Gamma(\chi_1^0 \rightarrow \tilde{G}\gamma)} &= \frac{\kappa_h}{\kappa_\gamma} \left(1 - \frac{m_h^2}{m_{\chi_1^0}^2}\right)^4 \end{aligned} \quad (\text{C.5})$$

$$\begin{aligned} \kappa_\gamma &= \frac{m_Z^4 s_W^2 c_W^2}{4M_1^4 M_2^4} (1 - \sin 2\beta)^2 (M_1^2 c_W + M_2^2 s_W)^2 + \dots \\ \kappa_Z &= \frac{1}{8} (1 - \sin 2\beta) + \frac{m_Z^2 \cos 2\beta^2}{8M_1 M_2 \mu} (M_1 c_W + M_2 s_W) + \dots \\ \kappa_h &= \frac{1}{4} (1 - \sin 2\alpha) - \frac{m_Z^2 \cos 2\alpha \cos 2\beta}{4M_1 M_2 \mu} (M_1 c_W + M_2 s_W) + \dots, \end{aligned} \quad (\text{C.6})$$

where $\tan 2\alpha = (m_A^2 + m_Z^2)/(m_A^2 - m_Z^2) \tan 2\beta$.

Thus, if there is an appreciable separation of scales $\mu, m_Z < M_{1,2}$, then $\kappa_{Z,h} \gg \kappa_\gamma$ and the decays to Zs and Higgses will dominate over the decays to photons. Note that because of the β^4 phase space factor, the decay rate to Z's will generally be slightly larger than the decay rate to Higgses.

C.3 Sparticle production at colliders

Finally, let us discuss briefly some differences between the production of bino vs. Higgsino NLSPs at hadron colliders. These will only be very preliminary remarks; a more detailed analysis will be contained in [35].

Assuming gluino and squark masses above ~ 1 TeV, the primary sparticle production modes at the LHC will be charginos and neutralinos produced from s-channel weak bosons. In this scenario, an increased Higgsino component can significantly alter the dominant production modes and cross sections.

First, let us consider the dominant production modes for bino vs. Higgsino NLSPs. Because the proton PDF's fall off so quickly with energy, the dominant production channels will generally be through the lightest modes. When $\mu \gg M_1, M_2$, the two lightest neutralinos and charginos are all gaugino-like, so we are only concerned with couplings of s-channel weak gauge bosons to gauginos. Hence, the relevant couplings are

$$q\bar{q} \rightarrow Z \rightarrow \tilde{C}_1^+ \tilde{C}_1^-, \quad q\bar{q}' \rightarrow W^\pm \rightarrow \tilde{C}_1^\pm \tilde{N}_2. \quad (\text{C.7})$$

Direct production of \tilde{N}_1 is suppressed because it is bino-like, and binos are neutral under electroweak.

Now let us contrast this with the situation for Higgsino NLSPs. When $\mu \ll M_1, M_2$, the two lightest neutralinos and charginos are all Higgsino-like and are all nearly degenerate around μ . Consequently, we care about the couplings of s-channel weak gauge bosons to Higgsinos, and the relevant channels are:

$$q\bar{q} \rightarrow Z \rightarrow \tilde{C}_1^+ \tilde{C}_1^-, \quad q\bar{q} \rightarrow Z \rightarrow \tilde{N}_1 \tilde{N}_2, \quad q\bar{q}' \rightarrow W^\pm \rightarrow \tilde{C}_1^\pm \tilde{N}_{1,2}. \quad (\text{C.8})$$

The first two channels are completely analogous to the two production channels for bino NLSP. The third channel, however, is an extra production mode, which is made possible because the two lightest neutralinos are nearly degenerate Higgsinos.

Finally, let us point out another difference between Higgsino and bino NLSPs which is apparent from (C.7), (C.8). Assuming the GUT relations amongst the gaugino masses, the dominant channels for bino NLSPs involve wino-like particles whose masses are $\approx 2m_{\text{NLSP}}$. On the other hand, for Higgsino NLSPs the dominant channels involve Higgsino-like particle whose masses $\approx m_{\text{NLSP}}$. Thus (at fixed NLSP mass) the sparticle production cross sections for Higgsino NLSPs are enhanced relative to those for bino NLSPs because the produced sparticles are lighter.

References

- [1] S.P. Martin, *A supersymmetry primer*, [hep-ph/9709356](#).
- [2] M. Dine, W. Fischler and M. Srednicki, *Supersymmetric technicolor*, *Nucl. Phys. B* **189** (1981) 575.
- [3] S. Dimopoulos and S. Raby, *Supercolor*, *Nucl. Phys. B* **192** (1981) 353.
- [4] M. Dine and W. Fischler, *A phenomenological model of particle physics based on supersymmetry*, *Phys. Lett. B* **110** (1982) 227.
- [5] C.R. Nappi and B.A. Ovrut, *Supersymmetric extension of the SU(3) \times SU(2) \times U(1) model*, *Phys. Lett. B* **113** (1982) 175.
- [6] M. Dine and W. Fischler, *A supersymmetric GUT*, *Nucl. Phys. B* **204** (1982) 346.
- [7] L. Álvarez-Gaumé, M. Claudson and M.B. Wise, *Low-energy supersymmetry*, *Nucl. Phys. B* **207** (1982) 96.
- [8] S. Dimopoulos and S. Raby, *Geometric hierarchy*, *Nucl. Phys. B* **219** (1983) 479.
- [9] M. Dine and A.E. Nelson, *Dynamical supersymmetry breaking at low-energies*, *Phys. Rev. D* **48** (1993) 1277 [[hep-ph/9303230](#)].
- [10] M. Dine, A.E. Nelson and Y. Shirman, *Low-energy dynamical supersymmetry breaking simplified*, *Phys. Rev. D* **51** (1995) 1362 [[hep-ph/9408384](#)].
- [11] M. Dine, A.E. Nelson, Y. Nir and Y. Shirman, *New tools for low-energy dynamical supersymmetry breaking*, *Phys. Rev. D* **53** (1996) 2658 [[hep-ph/9507378](#)].
- [12] G.F. Giudice and R. Rattazzi, *Theories with gauge-mediated supersymmetry breaking*, *Phys. Rept.* **322** (1999) 419 [[hep-ph/9801271](#)].

- [13] E. Poppitz and S.P. Trivedi, *Dynamical supersymmetry breaking*, *Ann. Rev. Nucl. Part. Sci.* **48** (1998) 307 [[hep-th/9803107](#)].
- [14] J. Terning, *Non-perturbative supersymmetry*, [hep-th/0306119](#).
- [15] Y. Shadmi and Y. Shirman, *Dynamical supersymmetry breaking*, *Rev. Mod. Phys.* **72** (2000) 25 [[hep-th/9907225](#)].
- [16] I. Affleck, M. Dine and N. Seiberg, *Calculable nonperturbative supersymmetry breaking*, *Phys. Rev. Lett.* **52** (1984) 1677.
- [17] I. Affleck, M. Dine and N. Seiberg, *Dynamical supersymmetry breaking in four-dimensions and its phenomenological implications*, *Nucl. Phys. B* **256** (1985) 557.
- [18] E. Witten, *Dynamical breaking of supersymmetry*, *Nucl. Phys. B* **188** (1981) 513.
- [19] A.G. Cohen, T.S. Roy and M. Schmaltz, *Hidden sector renormalization of MSSM scalar masses*, *JHEP* **02** (2007) 027 [[hep-ph/0612100](#)].
- [20] S.P. Martin, *Generalized messengers of supersymmetry breaking and the sparticle mass spectrum*, *Phys. Rev. D* **55** (1997) 3177 [[hep-ph/9608224](#)].
- [21] Y. Nomura and K. Tobe, *Phenomenological aspects of a direct-transmission model of dynamical supersymmetry breaking with the gravitino mass $m(3/2) < 1 \text{ keV}$* , *Phys. Rev. D* **58** (1998) 055002 [[hep-ph/9708377](#)].
- [22] K.I. Izawa, Y. Nomura, K. Tobe and T. Yanagida, *Direct-transmission models of dynamical supersymmetry breaking*, *Phys. Rev. D* **56** (1997) 2886 [[hep-ph/9705228](#)].
- [23] G.R. Dvali, G.F. Giudice and A. Pomarol, *The μ -problem in theories with gauge-mediated supersymmetry breaking*, *Nucl. Phys. B* **478** (1996) 31 [[hep-ph/9603238](#)].
- [24] S. Dimopoulos and G.F. Giudice, *Multi-messenger theories of gauge-mediated supersymmetry breaking*, *Phys. Lett. B* **393** (1997) 72 [[hep-ph/9609344](#)].
- [25] S. Dimopoulos, S.D. Thomas and J.D. Wells, *Sparticle spectroscopy and electroweak symmetry breaking with gauge-mediated supersymmetry breaking*, *Nucl. Phys. B* **488** (1997) 39 [[hep-ph/9609434](#)].
- [26] K. Intriligator, N. Seiberg and D. Shih, *Dynamical SUSY breaking in meta-stable vacua*, *JHEP* **04** (2006) 021 [[hep-th/0602239](#)].
- [27] K. Agashe, *Can multi-TeV (top and other) squarks be natural in gauge mediation?*, *Phys. Rev. D* **61** (2000) 115006 [[hep-ph/9910497](#)].
- [28] K. Agashe and M. Graesser, *Improving the fine tuning in models of low energy gauge mediated supersymmetry breaking*, *Nucl. Phys. B* **507** (1997) 3 [[hep-ph/9704206](#)].
- [29] H. Baer, P.G. Mercadante, X. Tata and Y.L. Wang, *The reach of Tevatron upgrades in gauge-mediated supersymmetry breaking models*, *Phys. Rev. D* **60** (1999) 055001 [[hep-ph/9903333](#)].
- [30] K.T. Matchev and S.D. Thomas, *Higgs and Z-boson signatures of supersymmetry*, *Phys. Rev. D* **62** (2000) 077702 [[hep-ph/9908482](#)].
- [31] H. Baer, P.G. Mercadante, X. Tata and Y.-l. Wang, *The reach of the CERN Large Hadron Collider for gauge-mediated supersymmetry breaking models*, *Phys. Rev. D* **62** (2000) 095007 [[hep-ph/0004001](#)].

- [32] SUSY WORKING GROUP collaboration, R.L. Culbertson et al., *Low-scale and gauge-mediated supersymmetry breaking at the Fermilab Tevatron Run II*, hep-ph/0008070.
- [33] D. Shih, *Spontaneous R-symmetry breaking in O’Raifeartaigh models*, *JHEP* **02** (2008) 091 [hep-th/0703196].
- [34] G.F. Giudice and R. Rattazzi, *Extracting supersymmetry-breaking effects from wave-function renormalization*, *Nucl. Phys. B* **511** (1998) 25 [hep-ph/9706540].
- [35] C. Cheung, A.L. Fitzpatrick, P. Meade and D. Shih, in preparation.
- [36] R. Barbieri and G.F. Giudice, *Upper bounds on supersymmetric particle masses*, *Nucl. Phys. B* **306** (1988) 63.
- [37] T. Kobayashi, H. Terao and A. Tsuchiya, *Fine-tuning in gauge mediated supersymmetry breaking models and induced top Yukawa coupling*, *Phys. Rev. D* **74** (2006) 015002 [hep-ph/0604091].
- [38] I.J.R. Aitchison, *Supersymmetry and the MSSM: an elementary introduction*, hep-ph/0505105.
- [39] M.E. Peskin, *Beyond the standard model*, hep-ph/9705479.
- [40] L. Ferretti, *O’Raifeartaigh models with spontaneous R-symmetry breaking*, *AIP Conf. Proc.* **957** (2007) 221 [arXiv:0710.2535].
- [41] L. O’Raifeartaigh, *Spontaneous symmetry breaking for chiral scalar superfields*, *Nucl. Phys. B* **96** (1975) 331.
- [42] R. Kitano, H. Ooguri and Y. Ookouchi, *Direct mediation of meta-stable supersymmetry breaking*, *Phys. Rev. D* **75** (2007) 045022 [hep-ph/0612139].
- [43] C. Csáki, Y. Shirman and J. Terning, *A simple model of low-scale direct gauge mediation*, *JHEP* **05** (2007) 099 [hep-ph/0612241].
- [44] H. Murayama and Y. Nomura, *Gauge mediation simplified*, *Phys. Rev. Lett.* **98** (2007) 151803 [hep-ph/0612186].
- [45] R. Kitano, *Dynamical GUT breaking and mu-term driven supersymmetry breaking*, *Phys. Rev. D* **74** (2006) 115002 [hep-ph/0606129].
- [46] R. Kitano, *Gravitational gauge mediation*, *Phys. Lett. B* **641** (2006) 203 [hep-ph/0607090].
- [47] M. Dine and J. Mason, *Gauge mediation in metastable vacua*, *Phys. Rev. D* **77** (2008) 016005 [hep-ph/0611312].
- [48] O. Aharony and N. Seiberg, *Naturalized and simplified gauge mediation*, *JHEP* **02** (2007) 054 [hep-ph/0612308].
- [49] A. Amariti, L. Girardello and A. Mariotti, *On meta-stable SQCD with adjoint matter and gauge mediation*, *Fortschr. Phys.* **55** (2007) 627 [hep-th/0701121].
- [50] S. Abel, C. Durnford, J. Jaeckel and V.V. Khoze, *Dynamical breaking of $U(1)_R$ and supersymmetry in a metastable vacuum*, *Phys. Lett. B* **661** (2008) 201 [arXiv:0707.2958].
- [51] N. Haba and N. Maru, *A simple model of direct gauge mediation of metastable supersymmetry breaking*, *Phys. Rev. D* **76** (2007) 115019 [arXiv:0709.2945].
- [52] A.E. Nelson and N. Seiberg, *R symmetry breaking versus supersymmetry breaking*, *Nucl. Phys. B* **416** (1994) 46 [hep-ph/9309299].

- [53] S. Raby, *Grand Unified Theories*, in PARTICLE DATA GROUP collaboration, W.M. Yao et al., *Review of particle physics*, *J. Phys.* **G 33** (2006) 1.
- [54] S.P. Martin and M.T. Vaughn, *Two loop renormalization group equations for soft supersymmetry breaking couplings*, *Phys. Rev.* **D 50** (1994) 2282 [[hep-ph/9311340](#)].
- [55] D.M. Pierce, J.A. Bagger, K.T. Matchev and R.-J. Zhang, *Precision corrections in the minimal supersymmetric standard model*, *Nucl. Phys.* **B 491** (1997) 3 [[hep-ph/9606211](#)].
- [56] B.C. Allanach, *SOFTSUSY: a C++ program for calculating supersymmetric spectra*, *Comput. Phys. Commun.* **143** (2002) 305 [[hep-ph/0104145](#)].
- [57] S. Dimopoulos, M. Dine, S. Raby and S.D. Thomas, *Experimental signatures of low energy gauge mediated supersymmetry breaking*, *Phys. Rev. Lett.* **76** (1996) 3494 [[hep-ph/9601367](#)].
- [58] S. Ambrosanio, G.L. Kane, G.D. Kribs, S.P. Martin and S. Mrenna, *Supersymmetric analysis and predictions based on the CDF $ee\gamma\gamma +$ missing E_T event*, *Phys. Rev. Lett.* **76** (1996) 3498 [[hep-ph/9602239](#)].
- [59] S. Dimopoulos, S.D. Thomas and J.D. Wells, *Implications of low energy supersymmetry breaking at the Tevatron*, *Phys. Rev.* **D 54** (1996) 3283 [[hep-ph/9604452](#)].
- [60] S. Ambrosanio, G.L. Kane, G.D. Kribs, S.P. Martin and S. Mrenna, *Search for supersymmetry with a light gravitino at the Fermilab Tevatron and CERN LEP colliders*, *Phys. Rev.* **D 54** (1996) 5395 [[hep-ph/9605398](#)].